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Conceptualising productivity measurement from a classical perspective

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Abstract:
Productivity seems an obvious concept: output per unit of input. Yet, when contextualised within alternative views of production and distribution, challenges across attempts at measuring it are far from trivial. The aim of this paper is to present and discuss some foundational concepts for measuring productivity from a classical perspective, as opposed to a more traditional standpoint. A key distinction is made between measuring productivity from the expenditure (or physical quantities) side and quantifying profitability from the value added (or income) side. Productivity is opposed to productiveness and commodity reduction is contrasted to price aggregation. After critically discussing the traditional standpoint of total factor productivity growth, this paper conceptually discusses the method of (growing) subsystems and the computation of production prices as analytical and empirical devices for measuring productivity and profitability in a multisectoral economy.

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Productivity has been a major ideology of institutions ever since medieval salvationist ideals ceded to modernist rationality.
Reich (2001, p. 40)

1. Introduction: why productivity?

Being trapped in history makes our analyses, in appearance, less general, less ‘scientific’, more speculative, full of counterfactuals, empty of timeless statements. Case-by-case detailed factual description or abstract reconstruction of the most remote past to the present day may appear as the only way to overcome the cruelty of the historical nature of economic phenomena.

The last couple of hundred years had been so little in comparison with thousands of years of man as a producer. Hence, political economy could be thought to be not very ambitious. Its object of study has been quite easily circumscribed to the analysis of phenomena of production and distribution in societies *after* the appearance of the capitalist mode of production.[1] Its theories are quite restricted in their timespan, and so are their projection onto the future.

By observing regularities, we look for principles. The organisation of collective labour is one of these. In this sense, the passage from value to reproduction prices can be analysed as a development in economic history, as a passage from simple to extended reproduction.[2] But in essence, I believe the crucial characterisation is the passage from an 'urban' to a 'capitalist' mode of production.

The 'urban revolution'[3] of the fourth millennium B.C. and the 'capitalist revolution' of the XVIII century are two of the most important transformations in the mode of production of human history. They both had, as a crucial determinant, the change in the social organization of collective labour and the specific use of the surplus produced by the community as a whole.[4]

Hence, the human determination to transform the organisation of collective labour is a deep cause in the appearance of the capitalist mode of production. As regards its enforcement, the delicate interplay with primitive accumulation[5] should be acknowledged. However, deep understanding is far from mechanical, as the laws to which the capitalist mode of production obey change continuously in their details, because the changing organisation of human labour in society feeds back into the root of the mode of production itself.

Wage labour, and individual ownership of the means of production, are probably the two main institutional mechanisms for the enforcement of a particular way of organising collective labour. And this is a general principle. But the details get continuously altered. And one of the details that changes in its long-period trends across time and space is the rhythm of surplus generating capacity of different societies, or, what amounts to the same thing, their productivity increases.

But productivity is an elusive concept. There abound meanings and uses (sometimes abuses) that place it as both cause *and* effect of economic growth and compositional changes in economic structure. It is my conviction that conceptual clarification can come about only by looking at the notion of productivity as a piece inside a comprehensive view of the process of production and distribution. As such, productivity analysis cannot be free of value judgement. Not even *physical* productivity, and clearly even less when surplus generating capacity is thought of in terms of profitability.

The aim of this paper is to provide a conceptual overview and discussion of alternative frameworks to define and measure productivity changes. Throughout, I adopt a classical perspective to economic analysis, trying to establish clear differences with traditional views as regards both theory and measurement. Hence, clarifying the meaning I attach to the 'classical' perspective is the place to start.

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[1] Though not necessarily being currently *under* the capitalist mode of production.

[2] As stated by Bródy (1970, p. 94): "The different price systems belonged to different historical layers for Marx".

[3] Of course the term 'revolution' is clearly a metaphor, its original meaning being "complete overturn of the relative positions of the various elements that constitute a system" (Liverani, 1998, p. 3). Processes of social and technological change are slow and gradual.

[4] For a development of the argument, see Gilibert (2010).

[5] In fact, "[t]he necessity of a primitive accumulation of capital, which society could invest in the structural conversion of its mode of production, was not only a Marxist theory. It was a general presupposition at the basis of 'classical' political economy" (Liverani, 1998, p. 7).

2. What do I mean by 'classical' perspective?

The classical standpoint to economic analysis has its roots in the works of the 'old classical economists', as intended by Sraffa (1960, Appendix D), with Quesnay's *Tableau Economique* as its first comprehensive scheme, followed by Marx's ([1885] 1978) schemes of 'Simple Reproduction' and 'Reproduction on an Expanded Scale', and extending into Leontief's (1937) input-output framework and Sraffa's (1960) system of production.[6]

The connections and interplay between these works are subject to long-standing debates, which shall not be dealt with here. Instead, the aim is to highlight only *some* features of a (modern) classical approach.

First and foremost, the characterising principle is the view of the economy as a circular process based on the notion of physical real costs, i.e., inputs are physically consumed in the process of generating the outputs that reconstitute the capacity to restart the very same process as inputs at the same (or enlarged) scale.

Essentially, "circularity stands here for repetitiveness" (Gilibert, 2006, p. 41, n. 16). Reproduction begins and returns to the same point, picturing the whole process as an ascending spiral whose diameter expands or contracts. In fact, as Sraffa would phrase it: "in a circular flow scheme 'The production of a thing has no real definite beginning – the inquiry leads us into infinite time backwards' (D3/12/7: 27)" (Kurz and Salvadori, 2005a, p. 497). Within the language of mathematics, circularity can be represented by eigensystems, which define prices and quantities in terms of themselves.[7]

Second, most of the interest lies in the analysis of those economies whose production exceeds its productive consumption, thus generating a surplus product, a set of commodities that must be allotted to different groups of individuals (classes) in society. Different uses of the surplus by the classes in society generate the income-expenditure loops of a circular process.

When commodities are produced by means of commodities, the notion of 'cost of production' as well as that of 'factor of production' may not survive close scrutiny, as the circular nature of the process of reproduction implies that produced means of production are both inputs and outputs, whose price or quantity as a 'cost' may not be known in advance of its price or quantity as a 'product'.

In fact, this circularity principle should not be confused with the marginalist view of the flow exchanges between 'product' and 'factor' markets, at the basis of general equilibrium theory. In this stylised view circularity is not straightforward, particularly when considering the notion of a market for 'capital' (or for the 'services of capital'): the 'quantity of capital' is supposed to be a quantitative magnitude "that can be measured independently of, and prior to, the determination of the prices of the products" (Sraffa, 1960, p. 9), with the rate of profits (interest) being its price. Hence, notional supply and demand schedules meet to determine the 'equilibrium' price and quantity of this 'factor', thus regulating – according to its 'relative scarcity' – income distribution between labour and capital (wages and profits).

But within the marginalist system, a clearing in the markets for 'factors' is crucial to the markets for 'products' as well, as "a demand schedule can only affect the price of the corresponding product to the extent to which it affects distribution" (Garegnani, 1983, p. 310). This is so because the functional relationship between commodity prices and final demand crucially depends on relative factor prices adjusting to the proportion of their employment as

[6] In no way does this brief comment aim to make an exhaustive enumeration of the works 'belonging' to the classical tradition of thought, which extends over centuries to this day.
[7] See Bródy (1970, pp. 84-94) for details.

inputs when demand for final output changes, i.e., the whole system depends on the mechanism of factor substitution.

It is, however, a well-established fact that the factor substitution mechanism cannot be sustained under reasonable assumptions for the representation of economic production.[8]

In contradistinction, when 'capital' is considered as a set of produced means of production – as it is from a classical perspective – it emerges that capital goods become part of the circular process of reproduction, so the view of them as an aggregate quantity measurable before the determination of prices is clearly untenable.

Furthermore, the rate of profits is *not* the 'price' for the 'quantity of capital'. By following Sraffa's interpretation of Ricardo ([1817] 1951):

> A method devised by Ricardo […] is that of singling out corn as the one product which is required both for its production and for the production of every other commodity. As a result, the rate of profits of the grower of corn is determined independently of value, merely by comparing the physical quantity on the side of the means of production to that on the side of the product, both of which consist of the same commodity (Sraffa, 1960, p. 93).

The difficulties of generalising this line of thought are dealt with by Sraffa (1960, pp. 22-23). The (uniform) rate of profits reflects a (possible) rule of distribution for the surplus product of the system. What should be clear is that, within the logical structure of classical analysis, the distribution of income between wages and profits has a 'degree of freedom', as has been rendered 'transparent' in Sraffa's (1960, Chapter IV) standard system.

Finally, when the demand and supply equilibrium theory of distribution is abandoned, even in the presence of variable returns, the determination of outputs entails them to be treated as independent variables *when* determining relative prices, and *vice-versa* (for details, see Garegnani, 1987).

Hence, another characterising feature of classical analysis is the separation between prices and quantities, i.e., between the value added and the expenditure side of the system. In fact, this aspect extends to empirical structures (input-output schemes, in particular) as well:

> Both the reproduction or *quantity and growth* aspect and the *price and distribution* aspect are dealt with, and it is made clear that input-output analysis is firmly rooted in the 'classical' tradition of economic thought (Kurz et al., 1998, p. xiv).

In this sense, from a classical perspective, relative prices "represent the exchange conditions between physical goods that make reproduction within a given technical (methods of production) and social (distribution) framework *possible*" (Schefold, 1989, p. 292).[9] Moreover, "with a surplus, prices are influenced by its distribution" (Gilibert, 2003, p. 34). Therefore, even an 'objective' theory of relative prices based on the technical conditions ensuing reproduction has a degree of freedom coming from outside the price equations, represented by the given rule of distribution of the surplus.

From the standpoint of classical analysis, it is possible to argue that the determination of relative physical outputs, i.e., the proportions of the system, starts from the consideration of a

[8] The marginalist process of 'input substitution' states that there exists "an inverse monotonic relation between the proportions of any two inputs and their relative prices" (Pasinetti, 1977, p. 390). For an explanation and critique, see Pasinetti (1977) but also Mas-Colell (1989, p. 505).

[9] Clearly, this conception of relative prices is in striking contrast with that of marginalist theory. Prices are not indices reflecting subjective individual preferences on the desirability of commodities in terms of their relative scarcities.

given (and measurable) set of commodity balances. However, its interpretation in terms of methods of production and activity levels of different processes is not unique.[10]

Therefore, from given empirical structures, different mappings to theoretical magnitudes can be attempted.

As to the present paper, the argument rests on the conception that observed tabulations of physical outputs in nominal terms contain both a self-replacement component and an expansion/contraction component, which need to be separated in *some* cases (though not in others).

What, in any case, should be clear is the role of the quantity system in the distinction of what re-enters the circular flow (and alters the productive capacity of the system) from what does not (and constitutes a truly final use). In this sense, we endorse the view that produced means of production are induced by the effective demand exerted for final 'consumption' commodities.[11] In other words, we conceive the 'accelerator' relation – the induced character of investment demand – as a mechanism at the basis of output determination. In fact, "[i]t is this *derived demand* aspect of investment goods, due to their being used as means of production, that is new and typical of production systems" (Pasinetti, 1981, p. 176).

However, this consideration of investment as induced by the growth trend of final effective demand should not be confused with the neoclassical 'inducement to invest'. In the latter case, demand for investment is made equal to the supply of savings through changes in the rate of interest. Under this view, current investment is seen as future consumption, so the interest rate is an equilibrating intertemporal price remunerating the consumption forgone today. Hence, the inducement to invest is the remuneration for the 'sacrifice' of waiting. Abstract 'waiting' becomes a 'factor' of production.

This neoclassical view raises three points for discussion. First, whether the interest rate regulates a long-period trade-off between present and future consumption (current investment). Second, whether this long-period trade-off actually exists. And third, whether abstract waiting can be considered a 'primary factor' of production on the same ground as labour.

The first point can be quickly addressed by noting that it presupposes the operation of the substitution mechanism (but from the dual quantity side). In fact:

> the standard one capital good case displays a monotonically decreasing relationship between consumption levels and the rate of interest […]. This does not generalize and it is now well understood that even with only two capital goods, a non-monotone association […] is possible (Mas-Colell, 1989, p. 506).

Hence, the interest rate *cannot* be seen as a regulator of intertemporal consumption levels, as it simply does not provide unambiguous 'price' signals.

The second point concerns whether higher investment demand necessarily implies lower consumption *levels*. This can be answered by noting that, from a modern classical perspective, capacity adjusts to demand, and investment generates the savings required by the system (not necessarily through a change in income distribution between wages and profits), so that an

[10] As argued in Garbellini and Wirkierman (2014), empirical matrices of intermediate consumption and gross fixed capital formation contain implicit expansion and contraction rates, because production requires time. Hence, the separation between activity levels and techniques is not always straightforward.

[11] This point is crucial. A capital good is not strictly determined on the basis of which industry is producing it, but also on the use that is made of it, i.e., on whether it re-enters the circular flow. Therefore, a machine that is exported is considered a final consumption commodity, while stocks of circulating capital produced in excess of self-replacement requirements that are devoted to expanding productive capacity in the following periods are considered *new* investment demand.

expansion of investment does not imply a necessary *decrease* in the level of consumption (though it will clearly alter its *share* in gross output).

As to the third point, it is particularly interesting that even some approaches that claim to be critical of marginal productivity theory endorse the view of 'waiting' as a primary factor of production:

> Because capital inputs are produced means of production, increases in efficiency due to advances in knowledge or increasing returns to scale will bring about increases in the output, input, and stocks of such capital inputs even when primary input flows remain constant [...] The question that then arises is: What is (are) the primary input(s) that is (are) behind the produced means of production? This is the question at the heart of the Cambridge capital controversy and has nothing to do with the question of the need or the feasibility of consistent aggregation (Cas and Rymes, 1991, pp. 8-9).

The answer to the question suggested by these authors reads:

> Individuals, supplying labour time, give up the immediate consumption of such time in exchange for the indirect consumption that the selling or nonimmediate consumption of such time avails them. Labour time sold is the measure of the labour input in productivity measures. Individuals, by forgoing present consumption and accumulating capital directly or indirectly through the bond and stock exchanges, are exchanging present for permanent consumption. *The accumulation of capital is the embodiment of the waiting individuals have done*. With technical progress, a given flow of waiting (or the forgoing of a given flow of present consumption) may be embodied in an increasing accumulation of capital that reflects the ever-increasing efficiency of waiting and results in higher levels of permanent consumption (Cas and Rymes, 1991, p. 11, emphasis added).

What this answer intends to do is *exhaust* value added (in this stylised case consisting of profits and wages) into as many 'primary' factors as there are components in it, and so justify claims to shares of value added on the basis of the subjective 'sacrifices' of working and waiting.

There is again a precise attempt to establish a 'natural' connection between an institutional phenomenon (the distribution of income) and a physical phenomenon (the accumulation of produced means of production). But, even more, this attempt is performed at the *individual* level (it is the single individual who works and waits).

Hence, the inverse monotonic relation between the real wage rate and the rate of profits is the trade-off between working (with the ensuing consumption) and waiting (with consequent saving). The social dimension of income distribution has disappeared (profits and wages are equally perceived by single individuals), and interpersonal income distribution comes to the fore, arguing that the distribution between wages and profits is not reflecting the dynamics of social classes but the interior struggle each individual has in deciding whether to work and consume or to wait and save resources.

The notion of *physical real cost* – that emerges from Sraffa's (1960) system of production – aims at establishing a sharp break with any notion of 'cost' conceived as the inducement to overcome the sacrifice of rendering resources available for their productive use. In this sense, the view of 'waiting' as a primary 'factor' serves the same purpose as the view of the endowment of a 'quantity of capital' (it just replaces stocks with flows, 'capital' by the 'use of capital').[12]

From a classical perspective this view is untenable: long-run accumulation is induced precisely by the growth of consumption and not by the abstinence from consuming and, while it

[12] In fact, in these analyses, the *labour* required for the expansion of productive capacity is considered as 'capital productivity' that accrues to 'waiting' (for example, see Gowdy and Miller, 1990, p. 594).

is correct that production takes time, this by no means entails that saving behaviour in the pursuit of interest rewarded as profit has a 'natural' right to become a 'primary' factor.

This theme leads to the role of the quantity of labour as an input in production. On this issue there are disagreements within the classical tradition, so these considerations cannot but be considered as highly personal. I believe one should carefully distinguish between labour inputs in the system of prices (i.e., the value added side) and the role of quantities of labour within the system of physical outputs (i.e., the expenditure side).

In a system of price equations, quantities of labour are explicitly considered as a component of prices through wage payments. In this sense, by using either a uniform wage rate or a vector of industry wage rates, the separation between quantities of labour and their rate of remuneration is an analytical construct.

In this context, as done by Sraffa (1960, p. 10), wage rate differentials could be applied to quantities of labour employed in every industry in order to render them homogeneous *as regards the process of pricing* (i.e., in order to be able to remunerate each unit of labour with a uniform wage rate w). But this should not be interpreted as a process that renders differing 'productivities' of labour homogeneous. Productivity plays absolutely no role in the application of wage rate differentials to quantities of employment.

In contradistinction, as regards physical outputs (i.e., in a set of commodity balances), quantities of labour do not explicitly appear in the system of equations. Their role in the analysis comes in when we acknowledge the necessary link between total employment and gross output (the human labour currently employed in all industries adds up to the total labour required to produce the gross output of the economy).

But it is possible to gradually modify the disaggregation of total gross output, starting from the sum of gross output by industry, then recognising indirect requirements to self-replace productive capacity (and so defining *net* output as the sum of new investment and final uses), until we account for the indirect requirements to expand productive capacity (and so defining *net* output consisting only of commodities for final uses). In each step, labour inputs by industry applied to these *different* direct and indirect commodity requirements redistribute total *co-existing* employment into as many parts as there are products, under each definition of *net* output.

Each of these (logical) parts constitutes a subsystem – in the sense of Sraffa (1960, Appendix A, p. 89) – and is identified by the single component of the net product to which it refers.

In each of these subsystems, the employment of all industries that is used for the production of the identifying *net* output commodity constitutes the labour content of this product. This is the logic adopted behind the notion of *labour content of commodities*. And under this conception, quantities of labour employed in each industry are all *equally necessary* to satisfy total reproductive requirements. Each industry is equally necessary, directly or indirectly, to produce each single commodity. Then, when applied to each decomposition of gross output, the vector of labour inputs (measured in units of full-time employment equivalents or total hours) is homogeneous, *as regards the determination of the labour content of commodities*.

Two points deserve clarification. First, while it is possible to interpret the notion of labour content as embodied labour, this can be done as long as it is clear that we always refer to living, co-existing, and concurrent employment.[13] The subsystem redistributes total direct employment into direct and indirect labour requirements to reproduce each different notion of net output, but it is always a logical construct. In no way do we consider embodied labour as a locked-in substance

[13] This view is clearly not new. In fact, on this specific aspect I follow Hodgskin ([1825] 1969). See also Milgate and Stimson (2009, pp. 225-231).

travelling through historical time (and methods of production) accumulated in commodities (durable means of production, for example).

The second point is that, from the above considerations, it emerges that zero profit production prices in terms of labour commanded are *not* necessarily equal to labour content of commodities. While a price system considers wage rate differentials, imported commodities, and taxes on products in the definition of a ‘zero profit’ price, the notion of labour content considers only domestically produced commodities (it is gross *output* and not total *supply* that is redistributed), excludes taxes on products, and considers the vector of labour inputs as homogeneous (therefore not allowing for a multiplication by a matrix of wage rate differentials). Hence, in all circumstances concerning *actual* systems (as differently from a theoretical model), these two magnitudes will *not* coincide.

In connection with this point, note that nowhere have I referred to the concept of ‘labour values’. The considerations just made should make apparent that it would deserve a more careful treatment than is customarily done, though this is not an aim of this paper.

However, one further related aspect should be mentioned, and this is the indispensable character of labour for the production process. In a system that does not admit full automation, i.e., labour is required directly or indirectly to produce at least one basic commodity,[14] labour inputs are indispensable.

It would be quite difficult to find a system where complete automation were possible. In such a hypothetical system, the notion of *labour content* would clearly lose its rationale, as the necessary link between total employment and gross output would be lost. As long as this is not the case, the distribution of total employment into logical parts reveals itself essential to assess the degree of *specialisation* within actual systems.

Given that, in purely abstract terms, the notion of physical productivity consists of measuring the rhythm of output generation per unit of input, it is possible to attach to each distribution of employment among subsystems a specific pattern of net product per unit of total (direct and indirect) subsystem labour, suggesting a consistent route for productivity analysis from a classical perspective

However, before further development of these ideas, and in connection with the dual view of prices and quantities at the root of classical analysis, it might prove interesting to clarify the difference between two very similar terms, with quite dissimilar meanings: productiveness and productivity.

3. Productiveness and productivity

The idea of productive, as opposed to unproductive, human labour can be traced back to the physiocratic distinction between ‘productive’ and ‘sterile’ classes in Quesnay’s *Tableau Economique*.[15]

In Physiocracy, a class was considered ‘productive’ or ‘sterile’ according to whether the labour it employed generated a net revenue or not.[16] As rent was the only net income of the system, whose origin was agricultural labour, the only ‘productive’ class was that of agricultural workers.

[14] In the sense of Sraffa (1960, p. 8).

[15] See Gilibert (1977, Chapter 4) for a clear exposition and discussion.

[16] Note the interesting meaning of the word *revenue*: “both the concept and the word came from France, where *revenu* is the past participle of *revenir*, to return” (Gilibert, 1987, p. 171). Here, revenue means the return to the starting point in a circular scheme.

But this represented a particular social configuration to which the general principle was applied: as a general principle, labour is productive as long as it generates a net revenue.

In fact, when Adam Smith treated the issue, he stated:

> There is one sort of labour *which adds to the value* of the subject upon which it is bestowed: there is another which has no such effect. The former, *as it produces a value*, may be called productive; the latter, unproductive labour (Smith, [1776] 1994, p. 360, emphasis added).

However, as with his treatment of value, a dual character appears: 'adding to the value' of the product is clearly in line with the physiocratic conception, though 'producing a value' has a further meaning:

> The labour of some of the most respectable orders in the society is, like that of menial servants, unproductive of any value, and *does not fix or realize itself in any permanent subject, or vendible commodity*, which endures after that labour is past (Smith, [1776] 1994, p. 361; emphasis added).

Productive labour, according to Smith, not only must add to the *value* of the product (i.e., workers are productive when the value of their production includes a net income), but it must also generate wealth, it must be materialised in an object. It is clear that both criteria need not always come together. In fact, the first criterion concerns exchange values generating a surplus (and this was the lead taken by, for example, Marx), while the second criterion concerns material characteristics of use values (being the lead taken by, for example, Malthus).

But there is an interesting point in Smith's first criterion. As noted by Gilibert (1977, p. 91), it is not the definition of productive labour that has changed with respect to Physiocracy but the social configuration to which the very same definition is applied. With the appearance of the capitalist mode of production, the greater the number of productive labourers generating a net income in the form of profits, the faster the rhythm of capital accumulation can be (as workers do not invest within this scheme). This is clearly reflected in Smith's title for Chapter III, Book II of *The Wealth of Nations*, from which the above quotations were taken: 'Of the accumulation of capital, or of productive and unproductive labour'.

Smith makes a value judgement as to the importance of the surplus generated by productive labour for the purpose of accumulation. In this he tries to make a connection between net income and its use as new investment, i.e., a connection between the value added (profits) and expenditure (investment) sides: the new social organisation of collective labour had to be reflected in a particular use of the surplus by the community.

In dealing with this issue, Marx notes:

> With the progressive accentuation of the co-operative character of the labour process, there necessarily occurs a progressive extension of the concept of productive labour, and of the concept of the bearer of that labour, the productive worker. In order to work productively, it is no longer necessary for the individual himself to put his hand to the object; it is sufficient for him to be an organ of the collective labourer, and to perform any one of its subordinate functions. The *definition of productive labour given above, the original definition*, is derived from the nature of material production itself, and it remains correct for the collective labourer. *But it no longer holds good for each member taken individually* (Marx, [1867] 1976, Chapter 16, pp. 643-644; emphasis added).

The *original definition*, which concerned the *individual* labour process,[17] stated that labour had been 'objectified' in the product the man had worked on, creating a *use* value.

But as the labour process becomes progressively co-operative, the productive character of each single worker has to be evaluated in the light of the *final* product that the 'collective labourer'

[17] See Marx ([1867] 1976, Chapter 7, p. 287).

obtains as a result: "[i]t is therefore to the collective labourer that one must refer in considering the labour embodied in a particular commodity" (Lippi, 1979, p. 1). And, as Marx observes, it is incorrect to apply the original criterion for the evaluation of each single worker. This is a particularly acute observation as to whether it is possible to quantify *individual* productivity.

In a circular flow scheme, each worker participates, directly or indirectly, in the production of every final commodity, though producing nothing by themselves. Therefore, which would be the numerator of any productivity measure for a single individual? Or, better, is it possible to conceive any aggregate productivity measure obtained as the sum of the 'productivities' of individual *microeconomic* agents? I believe the answer is in the negative as long as one retains coherence with a classical perspective.

In fact, it must be clear that the distinction between the *macro*-economic and the *micro*-economic must not be taken as a distinction between aggregate and disaggregated analysis. Inter-industry schemes represent macroeconomic phenomena, as they deal with the general interdependence of a system, no matter whether its basic unit of analysis consists of the elements of a set of disaggregated industries.

Marx, however, took one further step as to the consideration of the *productiveness* of labour, when he stated:

> Yet the concept of productive labour also becomes narrower. Capitalist production is not merely the production of commodities, it is, by its very essence, the production of surplus-value. The worker produces not for himself, but for capital. It is no longer sufficient, therefore, for him simply to produce. He must produce surplus-value. *The only worker who is productive is one who produces surplus-value for the capitalist* (Marx, [1867] 1976, Chapter 16, p. 644, emphasis added).

From this definition, Marx's third step[18] implies that the productiveness of labour comes from its social form, i.e., from the definition of what is net income *and* from its rule of distribution among social classes.

In any case, Marx's definition brings to the fore the essential character of productive labour, already present in Quesnay: labour is productive as long as it generates a net income (rent in Physiocracy, rent and capitalist profits in Marx). Hence, within the capitalist mode of production, the productiveness of labour is established by the criterion of *profitability*.

However, this opens up a crucial question. How is net income defined? It is possible to argue that neither Leontief (1951) nor Sraffa (1960) gave completely general definitions.[19]

As to Leontief, he clearly recognised an 'arbitrary' nature in the notion of net income, rendered explicit when discussing the aggregation of his system of input-output accounts:

> The process of consolidation, i.e., the reduction in the number of independent accounts, may proceed up to the point where the table is reduced to a single box. The *net* content of these completely unified accounts equals zero (Leontief, 1951, p. 16).

If what constitutes net income depends on the level of aggregation, the notion itself contains a degree of arbitrariness.

Instead, as to Sraffa, we read:

> The national income of a system *in a self-replacing state* consists of the set of commodities which are left over when from the gross national product we have removed item by item the articles *which go to replace* the means of production used up in all the industries. (Sraffa, 1960, p. 11, emphasis added).

[18] From the individual to the collective, from the collective to the historically determined capitalist labour process.
[19] See Gilibert (1987, p. 172) for details.

What happens outside the self-replacing state? There is no explicit answer. Moreover, Sraffa refers to the *value* of the net *product* (i.e., the physical surplus of commodities in excess of productive consumption) to establish a definition of net income. But this amounts to looking at net income from the expenditure side and not explicitly from the value added side. In fact, Sraffa develops the analytical construct of a wage partitioned into a 'productive consumption' component, a physically given wage basket included among the means of production, and a 'surplus' component, which participates in the net revenue of the system. The analytical construct of a partitioned wage could not be part of a purely 'objective' definition of net *income*.

However, the use of the net product in the definition of net income is quite interesting. Sraffa, though, does not develop the implications of alternative uses of this physical surplus, as his "investigation is concerned exclusively with such properties of an economic system as do not depend on changes in the scale of production" (Sraffa, 1960, p. iv). On the contrary, the distinction between the different uses of the physical surplus is precisely the starting point of Leontief's original view:

> But now we come up against the customary distinction between net and gross income, 'necessary' maintenance costs and 'free' profit. [...] The sense of surplus theory [...] is best understood if one enquires into the use of this 'free' income. The answer is: it either accumulates or is used up unproductively (Leontief, [1928] 1991, p. 209).

The key point I find behind this statement is that by shifting the focus from net *income* to net *product*, we are actually switching from profitability to productivity, from the value added side to the expenditure side of the system.

The productiveness of labour comes from its profitability. Instead, what remains essential for the definition of *physical* productivity is the separation between gross and net *product*, and what distinguishes this separation is *not* the distribution of the physical surplus among classes but separating what re-enters the circular process and alters the productive capacity of the system (i.e., *gross* investment) from what consists of a final use (consumption and exports).

Hence, instead of a rule of distribution, it becomes necessary to define a rule of accumulation (e.g., the accelerator relationship). And, instead of profits as net income, we need to identify consumption commodities as net product. Therefore, while for the analysis of profitability we should look at the system from the value added (relative prices) side, when the interest lies in analysing *physical* productivity, we should look at the economy from the expenditure (or product balances) side.

In fact, if we consider the simplest algebraic representation of the economy, gross output (X) can be alternatively seen as:

$$X = U + W + \Pi \qquad \text{(value added side)}$$

$$X = U + C + \mathrm{I} \qquad \text{(expenditure side)}$$

with U as the means of production that go to replace productive capacity, W as wages, Π as profits, C as consumption, and I as *new* investment.

Now, to define net income and net product, respectively, we may proceed as follows:

$$\Pi = X - U - W \qquad \text{(net income)}$$

$$C = X - U - \mathrm{I} \qquad \text{(net product)}$$

In this way, the labour that participates in the generation of profits (Π) is seen as productive labour, and the labour required to reproduce final consumption commodities (C) is a possible measure for the physical productivity of the system.[20]

The distinction between productiveness (i.e., profitability) and physical productivity becomes essential for an adequate conceptualisation of the measurement of productivity changes.[21] On many occasions such a distinction has been submerged, resulting in conceptual confusion. The traditional (marginalist) measurement of productivity changes is clearly no exception to this conceptual blur.

4. Traditional measurement of productivity changes

While Sraffa defined net income as the *value* of the net product, traditional analysis of technical change – based on marginal productivity theory – proceeded precisely in the opposite way, assigning to each component of net income (i.e., of value added) a *physical* interpretation.

Traditional analysis starts from an accounting identity for value added, disaggregated into as many 'primary factors' as there are components in it. Consider the simplest case, in which there are only two such components, profits and wages. Then, using the notation previously introduced, we have:

$$X - U = \Pi + W$$

The first step consists in separating each component into the product of a quantity element by its corresponding rate of remuneration. Consider the simplest case, in which there is a homogeneous 'quantity of capital' K, with a rate of remuneration r, and a homogeneous quantity of labour L, with a rate of remuneration w:

$$X - U = pQ,\ \ \Pi + W = rK + wL \quad \text{and therefore} \quad pQ = rK + wL$$

The second step consists in adopting a standard of value, generally establishing the price of a unit of value added equal to one; then $p = 1$. In this case, the passage of time implies that:

$$\frac{d\ln Q}{dt} = \left(r\frac{K}{Q}\right)\left(\frac{dK}{K}\frac{1}{dt}\right) + \left(w\frac{L}{Q}\right)\left(\frac{dL}{L}\frac{1}{dt}\right) + \left(\frac{dr}{dt}\frac{K}{Q} + \frac{dw}{dt}\frac{L}{Q}\right) \tag{1}$$

At the same time, the production process is represented by, for example, $Q = F(K, L, t)$. Then, the passage of time implies that:

$$\frac{d\ln Q}{dt} = \frac{d\ln F(K,L,t)}{dt} = \left(\frac{\partial F}{\partial K}\frac{K}{Q}\right)\left(\frac{dK}{K}\frac{1}{dt}\right) + \left(\frac{\partial F}{\partial L}\frac{L}{Q}\right)\left(\frac{dL}{L}\frac{1}{dt}\right) + \frac{\partial F}{\partial t}\frac{1}{F} \tag{2}$$

Now, in order to relate equation (1) with equation (2), it is common practice to assume:[22]

[20] In this context, it would be provocative to ask: what is the productiveness of labour in an economy without profits? And, at the same time, what is the physical productivity of labour in an economy without consumption? Two certainly interesting questions.

[21] Note how subtle this distinction might sometimes be: Sraffa's (1960) book has a very slim index, giving privilege to truly essential points. We do not find an entry for 'productivity' or for 'profitability', not even one for 'net income', though (with reference to the discussion of negative employment multipliers in the presence of joint products) we find the following entry: 'Negative productivity of labour, not necessarily unprofitable'.

[22] See Rampa (1981b, pp. 109-112) for an exposition with interesting (critical) remarks.

1. The actual economy is immersed in a competitive general equilibrium, in the sense that primary factors are paid their 'marginal products' for a given $F(K, L, t)$, i.e., $r = \partial F/\partial K$ and $w = \partial F/\partial L$.
2. The share of each primary factor in value added is constant during the comparison, i.e., $\omega_K(t) = \omega_K = r(K/Q)$ and $\omega_L(t) = \omega_L = w(L/Q)$.
3. The autonomous time trend of value added in constant prices $(\partial F/\partial t)(1/F)$ is independent of K/L (see Solow, 1957, p. 313), and is obtained by keeping constant all (primary) inputs.

Then, if we call $\rho_{tfp} = (\partial F/\partial t)(1/F)$ the rate of total factor productivity (TFP, hereinafter) growth, two empirical strategies emerge from the application of the assumptions above to (1)-(2):

$$\rho_{tfp} = \frac{d\ln Q}{dt} - \omega_K \frac{d\ln K}{dt} - \omega_L \frac{d\ln L}{dt} \tag{3}$$

$$\rho_{tfp} = \omega_K \frac{d\ln r}{dt} + \omega_L \frac{d\ln w}{dt} \tag{4}$$

Expressions (3) and (4) represent the primal and the dual approaches to TFP growth measurement, respectively.

The logic behind the primal measure (3) runs as follows: under constant returns to scale, assumptions (1) and (2) above identify a production function invariant in time, which is applied to new inputs. If the 'theoretical' output obtained is different from that effectively measured, $\rho_{tfp} \neq 0$, and this will be interpreted as a *shift* in the production function. On the contrary, if $\rho_{tfp} = 0$, changes in value added in constant prices will be interpreted as resulting from the movement *along* a given (and invariant) production function.

Hence, ρ_{tfp} represents 'technical change' as long as the difference between output and (primary) input growth can be *interpreted* as a shift in the production function, and this is accomplished by *assuming* that 'physical' value added in constant prices is obtained according to $Q = F(K, L, t)$ and the actual economy is in a competitive equilibrium. In fact:

> *If* a production function has constant returns to scale and *if* all marginal rates of substitution are equal to the corresponding price ratios, a change in total factor productivity *may be identified* with a shift in the production function (Jorgenson and Griliches, 1967, p. 250, emphasis added).

Complementarily, the logic behind the dual measure (4) is based precisely on the equality between factor prices and marginal products, so that TFP growth measures how shifts in the production function accrue to 'capital productivity' (measured by *r*) and 'labour productivity' (measured by *w*).

The dual approach to growth accounting has not been explored as much as the primal one (though it is already explicit in, e.g., Jorgenson and Griliches, 1967, p. 252, eq. 4). Understandably, they should both give *similar* results if 'factors' are paid their 'marginal products' and aggregation into a 'quantity of capital' poses no serious problems. However, this has not been the case in studies like that of Hsieh (1999), where significant differences have been found in some estimates. I would find it rather difficult to attribute all the discrepancies to index number problems or measurement errors.

In any case, reconsider the whole structure of the approach, beginning from its definition:

> The rate of growth of total factor productivity is *defined* as the difference between the rate of growth of real product and the rate of growth of real factor input. The rates of growth of real

> product and real factor input are defined, in turn, as weighted averages of the rates of growth of *individual* products and factors (Jorgenson and Griliches, 1967, p. 250, emphasis added).

TFP growth is defined as the difference between output and input growth. It is an *effect*, a consequence of two causes (outputs and inputs). How can this be reconciled with TFP *explaining* output growth, as argued by modern growth theory? Only by assuming that the actual system behaves as the total differential of $Q = F(K, L, t)$ in a competitive equilibrium for each period under comparison. So growth theory is taking for granted what it should empirically test, i.e., whether the system does behave competitively, and 'factors' are remunerated accordingly.[23]

As noted in Rampa (1981b, p. 111), a related point concerns the stringent character of assumption 2 above. If, in order to identify an invariant production function, it is necessary to keep factor shares (ω_K, ω_L) constant, there is an index number problem that gives rise to a relevant degree of arbitrariness. Changing the single (or average) period from which to take factor shares may result in considerably different TFP growth rates.

Note, moreover, that the definition just stated explains how the approach proceeds by aggregating individual products and factors. Disaggregation can go on until the single microeconomic agent is reached, and her/his/its productivity will be given by the remuneration she/he/it obtains for the services provided. But this can be linked to the discussion about the 'collective labourer' previously introduced. Is it possible to measure physical productivity for a single individual? I believe not. But it is not a problem of insufficient data, it is a matter of principle.

In fact, while it is possible to measure the *productive* character of a single employment unit (it is enough to see whether it contributes to net income, e.g., profits), it is not possible to quantify its physical productivity: in a society with a considerable degree of division of labour, a single agent will contribute (directly or indirectly) to the production of all products, though producing nothing completely by itself.

But, even granted the logic of the definition, there are still rather important points for discussion.

For example, TFP growth intends to measure disembodied technical change, as capital accumulation is separated from the 'speed of technical rationalization'. But, if changes in technology also occur in the production of capital goods (for details, see Pasinetti, 1959, p. 270), their reproduction implies a different use of resources according to the general state of efficiency (the economy-wide TFP rate). And therefore:

> measured capital deepening is not necessarily an independent contributor to growth, rather it is contingent upon prior shifts in the production function, that is to say the rate of technical progress (Metcalfe, 2002, p. 4).

Technical change may be *embodied* in new vintages of capital goods, so the TFP growth rate, as conventionally measured, is not even reflecting the comprehensive effect of producing commodities with *better* commodities.

This point raises an issue on the definition of output and input and the representation of the production process in traditional measurement. Note that it is assumed that value added in constant prices is the disaggregated measure of 'physical' output, the 'quantity of capital' is a non-produced primary input, and production is pictured as a "one-way avenue that leads from 'factors of production' to 'consumption goods"' (Sraffa, 1960, p. 93). These choices have important consequences on the results.

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[23] In fact, in this context, it should be remembered that "decomposition does not imply anything about causality" (Balk, 2010, p. S234).

In a multisectoral economy, value added is a residual rather than an independent entity, both according to surplus theory and to the System of National Accounts.[24] From the equations above, when the means of production for self-replacement are deduced from gross output, an artificial separation between a price and volume component is performed: $X - U = \mathrm{pQ}$. If X and U are different composite commodities, then their difference (value added) cannot be an independently measurable composite commodity to which a price index is attributed. Value added is the *effect* of two causes (X and U), so it is difficult to maintain that it has a physical meaning in itself.[25]

Furthermore, taking value added as a disaggregated measure of output does not give a comprehensive picture of the production *process*. Production is circular, and commodities can be given final as well as intermediate uses. For productivity measurement, it is crucial to know the ratio between gross output and final uses, as an economy with a higher ratio would need more gross output to produce the same physical surplus, in the presence of input saving due to technical change. This is not necessarily captured at a disaggregated level by a net income measure such as value added.[26]

Note, then, that value added cannot be a coherent *disaggregated* measure of physical output, and the 'quantity of capital' cannot be a coherent *aggregated* physical measure of inputs. Hence, it is not a matter of aggregation or disaggregation that leads TFP growth to run into trouble.

But even granting a physical interpretation to value added, the common functional forms used for empirical measurement in traditional analysis are not exempt from an essential dimensionality critique, as advanced by Bródy (1970, pp. 95-96). Consider a traditional function like: $\mathrm{Q} = AK^{\alpha}L^{1-\alpha}$, where Q is measured in flow of money (M) per year (T), K is measured as a stock of money (M), and L is measured in man years ($H \times T$). Then, we have: $M/T = A \times M^{\alpha} \times (H \times T)^{1-\alpha}$.

Now, what is the dimension of A (i.e., the TFP parameter)? If $\alpha = 1$, $M/T = A \times M$, so $A = 1/T$ (the reciprocal of TFP is measured in time units, 'productivity' as *time*). If $\alpha = 0$, then $A = M/(H \times T^2)$, which could be reordered to mean money flow per man year. And what if $0 < \alpha < 1$ (which is the normally assumed case)? TFP parameter A does not have any definite meaning. So, what is the *dimension* of TFP? It remains an open question, unless one abandons its purely physical interpretation, based on production functions, and admits that TFP is an accounting 'residual', simply reflecting 'real cost reductions'.[27]

As with the confusion in the distinction between micro/macro and aggregate/disaggregated, it is generally thought that TFP is a complete ('total') measure as against 'partial' measures that consider only, e.g., labour productivity. But it is perfectly possible to derive measures of total labour productivity as well.[28] The confusion has its origins in the way TFP (or the 'residual') has been conceived:

> In whichever form, the measured residual typically accounted for an important fraction of the observed output growth, quite often half or more.
> This result came as a surprise to the profession, though perhaps less so to those who reached it, or something very like it, *by an alternative route*. They were the people who came at the problem *out of a tradition of measuring labor productivity*, and at some point complemented output per worker with a measure of output per unit of capital, and *finally joined the two* to create a measure of total factor productivity (TFP) (Harberger, 1998, pp. 1-2, emphasis added).

[24] See Wirkierman (2022a, p. 496, footnote 1) and the discussion in Wirkierman (2012, Chapter 3) for details.
[25] On this point, see Meade (2010).
[26] See Rampa (1981b, p. 111) on this point.
[27] For example, see Harberger (1998, p. 3).
[28] See Wirkierman (2012, Chapter 2) for a discussion and Garbellini and Wirkierman (2014).

The 'tradition of measuring labour productivity' (measured as industry *value added* per unit of labour) was conceptually focused on the *productiveness* of labour, not on its physical productivity. The 'total' character of TFP comes from the conception of production and distribution according to marginal productivity theory. If capital and labour are the two prominent non-produced primary factors, a measure of *productiveness* has to account for the net income they both generate. But the qualification 'total' is clearly contingent on a *purely theoretical* choice.

What should be clear is that what distinguishes a 'partial' from a 'total' physical *productivity* measure is not the differentiated components in net income but whether the general interdependence of the system is being taken into account or not. A total measure of labour productivity, i.e., the computation of the labour content of *all* (direct and indirect) commodity requirements to reproduce the net output (possibly expanding productive capacity), considers capital goods as well, accounting for their circulation as inputs and outputs through the 'convoluted' network of input-output transactions.

And in this sense, the input-output literature on TFP growth measurement is both relevant and insightful.[29] Net income and net product are, in most cases, clearly differentiated, and the latter is used to derive aggregate measures (for example, see Wolff, 1994, p. 81, eq. 3). But, while interdependence is accounted for as regards circulating capital inputs, fixed capital is still conceived as a primary factor of production in most cases.[30]

In any case, I think the crux of the matter boils down to the lack of an explicit distinction between physical productivity and productiveness (i.e., profitability). If the derivation of a TFP figure (be that using production functions or by means of input-output schemes) starts from a theory of value added, it cannot lead to adequate measurement of disaggregated physical productivity. In fact, even abandoning some stringent assumptions of marginalist production theory, and adopting a sceptical view of the residual simply as a reduction in real costs, the confusion about productiveness and productivity remains:

> A simple TFP measure *for firms* with multiple outputs and multiple inputs is to look at the *profitability of a firm*, defined as the revenue of the firm divided by its input cost. […] [A] strict comparison […] is difficult since the output and input prices faced by these firms are different. The only option here is to adjust the value aggregates […] for differences in price levels (Coelli et al., 2005, pp. 62-63, emphasis added).

However, to privilege a physical interpretation of productivity does not mean in the least to deny the importance of profitability changes in the evaluation of the effects of technical change in employment, output proportions, and relative prices. What must be clear is the separation in concepts and terminology that is required to distinguish cause from effect.

5. Subsystems and prices of production

The distinction between physical productivity and profitability is not only established by the alternative reference to the expenditure or value added side of the economy as a starting point, but also to the unit of analysis to which each notion refers.

[29] See, for example: Wolff (1985), Galatin (1988), Wolff (1994), ten Raa (1994), Casler and Gallatin (1997), as well as Wirkierman (2012, Chapter 3).
[30] See Peterson (1979, pp. 218-219) and Wolff (1994, pp. 84-86) for some exceptions.

In fact, the economic system can be disaggregated in industries, producing a product mix of outputs, but it can also be disaggregated in self-replacing and growing subsystems[31] that produce a single kind of *final* commodity. While profitability measures are computed at the industry level, disaggregated physical productivity measures are formulated with the subsystem in mind.

5.1. Production prices and eigensystems

Computable production prices can be conceived of as prices implicit in the (medley of) technique(s) in use in the economic system. They represent exchange ratios satisfying necessary conditions for reproduction under a *given* rule of distribution of the surplus (e.g., a uniform profit factor is computed in proportion to capital advanced).

The technique in use can be thought to be contained in measurable empirical structures, like input-output schemes. And empirical structures reveal changes in techniques when nominal figures can be separated into a volume growth and a price component. In this way, production prices function as aggregators applied to the quantities actually produced to measure changes in inputs and outputs, thus allowing to quantify the changing surplus (in value terms) between revenues and outlays by industry (i.e., changes in profitability due to technical changes for a given distributive configuration).

An important remark on method is in order. Note that, by interpreting computable classical production prices as a *possible* set of aggregators,[32] no presumption of descriptive or predictive power is attributed to them. By so doing, the aim is to separate two fields of inquiry.

In this sense, computable prices could be conceived as one answer to the following problem, posed by Sraffa:

> The problem is that of *ascertaining the conditions of equilibrium of a system of prices and the rate of profits, independently of the study of the forces which may bring about such a state of equilibrium*. Since a solution of the second problem carries with it a solution of the first, that is the course usually adopted in modern theory. The first problem however is susceptible of a more general treatment, independent of the particular forces assumed for the second; and in view of the unsatisfactory character of the latter, there is advantage in maintaining its independence (D3/12/15: 2; emphasis added) (Kurz and Salvadori, 2005b, p. 433).

In fact, among the first computations of prices of production, the works of Hejl et al. (1967), Kyn et al. (1967) and Sekerka et al. (1970) – with an origin in the dynamic input-output price model – had the aim of comparing different price *norms* applied to the same technique in use.

Since then, however, computable production prices have been used for multiple purposes; for example, the empirical assessment of capital theory paradoxes, going from Krelle (1977) to Han and Schefold (2006). In connection with this are the degree of (non-) linearity of wage-profit schedules for different standards of value and the study of the deviation of labour values (empirically defined in a variety of ways) from production prices and market prices.[33] A related

[31] Throughout the paper, the terms self-replacing subsystem and vertically integrated sector are used interchangeably, and the same applies for the terms growing subsystem and vertically hyper-integrated sector. While the notion of self-replacing subsystem was introduced by Sraffa (1960, Appendix A, p. 89), its compact representation as a vertically integrated sector was introduced by Pasinetti (1973), and its extension to growing subsystems or hyper-integrated sectors was introduced by Pasinetti (1988). See Garbellini and Wirkierman (2014) for a detailed discussion.

[32] Computable prices may be considered to be one among the many possible *norms*, in the sense of Pasinetti (1981, p. 127n).

[33] See Marzi (1993) for a review of these lines of inquiry. The crucial points under discussion are: (a) the empirical plausibility of the neoclassical parable of a surrogate production function, (b) the regularity in relative price changes as

use has been to study computable prices under the assumption of heterogeneous rates of profits between industries (though generally maintaining a fixed *relative structure* of profit rate differentials).[34]

For the purpose of this paper, the interest lies in the use of computable prices to measure the effects of technological progress on the distributive possibilities of the system, i.e., computation of changes in profitability induced by changes in the technique in use, for each given distributive configuration. Operationally, this takes place by assessing the shape of 'wage-profit curves'. Among empirical works with this aim, it is possible to mention:

1. The studies of Marzi and Varri (1977) (using only circulating capital) and Marzi (1982) (including also fixed capital) for the Italian economy, assessing the Harrod-neutral character of changes in techniques.
2. The works of Ozol (1984, 1991) for Canada and the US, concluding on the 'cost reducing automation' character of technical change.
3. The study of Leontief (1986) for the US, introducing a linear program to find the most profitable (for a given real wage rate) technique, at each feasible value of the rate of profits.
4. The study by Marzi (1994) for the Italian economy (using only circulating capital), which relates the convexity of the wage-profit schedule (as each final commodity alternatively becomes the standard of value) to the capital intensity of the (balanced) growth subsystem associated with each final product.
5. The work of Degasperi and Fredholm (2010), proposing the area under alternative wage-profit schedules (for a common *numéraire* commodity) as an indicator of the degree of technical progress across time and space, with an application (using only circulating capital) to selected OECD economies.

All of the above-mentioned studies have in common a classical awareness that is applied to input-output data. Some related studies analyse technical change by means of wage-profit curves, which are built only with national accounts data by industry (for example, see Michl, 1991; Ferretti, 2008).

When Leontief (1951) noticed that, by consolidating input-output accounts, it was possible to obtain zero *net* income, he connected each income source with a specific use of expenditure. Clearly, the view of an economy with both zero net income *and* zero surplus product corresponds to a closed system, where *all* income-expenditure loops are endogenous.

In such systems, there is complete duality between prices and quantities and between income and expenditure (there are no final uses), and the notions of productivity and profitability acquire a dual character as well. Mathematically, the structure of closed systems is usually formulated as an eigen-problem, where the maximum (in modulus) eigenvalue summarises the system's surplus both in physical and value terms.

Starting from the seminal work of Bródy (1970), different studies have *conceptualised* the measurement of productivity increases as an eigenproblem.[35] Though very appealing from an

income distribution is altered, and (c) the empirical plausibility of a pure labour theory of value. Note that all three cases imply a *descriptive/predictive* role for the notion of classical production price.

[34] See Flaschel (2010, Part II, Chapter 8, section 8.5) for details.

[35] For example, Buccellato (1990) has used the left and right eigenvectors associated with the maximum eigenvalue to compute shares in production according to 'standard proportions' or revalue industries according to their associated 'standard prices', in order to detect market over- and under-valuation. In another study, Aulin-Ahmavaara (1999, p. 358) develops (only in theoretical terms) her 'fully effective rate' of productivity change, by assuming that all inputs are (re)produced (including labour).

analytical point of view, its empirical implementation is not straightforward. To adequately separate (statistical) prices from volume growth, it is necessary to neglect the presence of imported commodities (which represent a full matrix as opposed to a single column vector of exports) as well as the presence of taxes on products and production. Moreover, it is necessary to deal with an essential empirical disarray in both wages ≡ consumption and profits ≡ investment theoretical identities.

In any case, the formulation of an eigensystem for the complete 'augmented' matrix and the study of its spectral properties should not be confused with the computation of the maximum eigenvalue of inter-industry flow matrices to assess the intensity of use of intermediate inputs (generally circulating capital).[36] In these cases, the spectral properties serve as a summarising device (with eigenvectors as a particular system of weights for aggregating rows and columns) and not as a comprehensive measure of surplus of self-contained closed systems.

5.2. Subsystems

The measurement of disaggregated physical productivity of labour with reference to the subsystem as a unit of analysis has its origin in considering jointly Leontief's (1953) computations of "the roundabout, as compared with the direct, effects which the changes in the input structures of various industries have on the *over-all productivity of labour*" (Leontief, 1953, p. 39; emphasis added), and Sraffa's (1960, Appendix A, p. 89) constructive algorithm to build the logical device of a subsystem.[37]

Even before the appearance of the subsystem, Pasinetti (1959) (in reply to Solow's (1957) contribution) had already noticed the need to account for the reproducible character of capital goods, measuring them in terms of final commodity-specific units of capacity, in order to derive physical measures of productivity changes.[38] After the subsystem had been devised, Pasinetti (1963, Chapter V) introduced the first algebraic and conceptual relations connecting industries with vertically integrated sectors (i.e., self-replacing subsystems) for the study of technical progress.

Moreover, Gossling and Dovring (1966) presented the first empirical application of productivity measurement adopting the subsystem as a unit of analysis. Gossling (1972) provided a comprehensive empirical study of productivity growth by subsystem, including a comparison with traditional 'partial' and 'total' factor measures.[39]

The crucial idea behind the subsystem is its degree of autonomy. By repartitioning the whole *row* vector of gross outputs and the matrix of intermediate uses by industry into as many logical parts as there are components in the *column* vector of final uses by commodity, all means of production, labour, and outputs are redistributed into each of these parts, according to their contribution as a supporting industry to the activity which produces the final commodity.

[36] See, for example, Rampa and Rampa (1982) (including also fixed capital and imported commodities) and Marengo (1992).

[37] A related approach to the measurement of direct and indirect labour requirements, explicitly recognising its meaning as a 'productivity' index, has been advanced by Vincent (1962), who suggested the term "productivité intégrale du travail" (total productivity of labour). See Gossling (1972, pp. 52-54) for a discussion.

[38] See Garbellini and Wirkierman (2023) for an exposition of the debate between Solow and Pasinetti.

[39] In a related article, Gossling (1974) complemented his study considering the open economy, i.e., direct and indirect requirements of imported commodities.

The redistribution of commodities in association with others, as an alternative to the aggregation of industries, is thoroughly discussed by Leontief (1967), who noticed that aggregation and reduction were two strategies to deal with too detailed empirical structures:

> Aggregation, i.e. summation of essentially heterogeneous quantities, is one of the two devices that the economist uses to limit the number of variables and functional relationships in terms of which he describes what he observes. The other is reduction, that is, elimination of certain goods and processes (Leontief, 1967, p. 419).

In a fundamental contribution, Pasinetti (1973) established explicit connections between the subsystem and the logical device of vertical integration, i.e., the reduction of some commodities in terms of others. By introducing a compact algebraic representation of a self-replacing subsystem, as the result of vertically integrating co-existing total employment and capital goods, it became possible to work with alternative representations of the same technique in use, either in direct terms (direct labour and direct productive capacity) or in vertically integrated terms (vertically integrated labour and productive capacity).

But, even though it dealt with the case of balanced growth, Pasinetti's (1973) construct remained essentially static, in the sense of representing only self-replacing subsystems. New investments were still included in the net output, so part of the physical surplus of industries producing capital goods still needed to be exchanged between (or redistributed among) these self-replacing sectors, in order for each of them to expand their *commodity-specific* productive capacity. This clearly posed difficulties for the degree of autonomy of the self-replacing subsystem.[40]

Thus, in the context of a dynamic economy, Pasinetti (1988) introduced the logical device of vertical hyper-integration in explicit association with the notion of a *growing* subsystem. The key difference is that *gross* investment to self-replace and expand commodity-specific productive capacities is redistributed among industries according to their reproduction requirements (which now include expansion/contraction) when the reduction process is performed. Therefore, investment becomes fully induced by the growth of effective demand for final uses.[41]

Among the different works either applying the concept of total labour requirements or explicitly adopting a self-replacing subsystem perspective, it is possible to mention:

1. The study by Gupta and Steedman (1971) for the UK, in which direct and total labour requirements are computed, leading the authors to conclude that "total (or system) labour use falls but less rapidly than direct labour use" (Gupta and Steedman, 1971, p. 32).
2. The empirical studies by Rampa (1981a) and Rampa and Rampa (1982), the theoretical considerations of Siniscalco (1982), and the work by Fredholm and Zambelli (2009), which explicitly adopt Gossling's (1972) operator to map between industries and self-replacing subsystems.
3. The studies by Ochoa (1986), De Juan and Febrero (2000), and Flaschel (2010, Part I, Chapter 3, pp. 63-68), connecting total labour productivity to labour content of commodities through labour values.
4. The theoretical considerations by Seyfried (1988), who explicitly separates vertically integrated labour productivity from vertically integrated labour rentability, this latter concept measuring "how much labour must be disposed to produce one unit of output […] if beside the

[40] The idea of this subtle but essential point is due to Garbellini (2010, pp. 48-51). The reader is referred to this source for a clear exposition and discussion.
[41] See Garbellini and Wirkierman (2014) for a detailed exposition.

reproduction of capital the capital owners can claim part of the product as profit" (Seyfried, 1988, p. 172).
5. The studies by Milberg and Elmslie (1992), Elmslie and Milberg (1996), and Dietzenbacher et al. (2000), which adopt an input-output approach to study cross-country convergence of labour productivity at the vertically integrated level.

Further developments of this line of research have devised, discussed, and empirically implemented sectoral (and aggregate) productivity measures in terms of *growing* subsystems (or vertically *hyper*-integrated sectors):

1. The study by Garbellini and Wirkierman (2014), establishing a direct correspondence between Supply-Use Tables and Pasinetti's (1973, 1988) theoretical magnitudes, making an explicit price-volume separation and exploring the possibility of empirically separating growth from the technique in use.
2. The contribution by Brondino (2019), applying the (hyper-)subsystem approach to the Chinese economy, finding that subsystems with the highest productivity performance had been targeted by industrial policy.
3. The work by Wirkierman (2022a), comparing hyper-integrated productivity dynamics amongst advanced industrial economies, finding that productivity gains accruing to wages were amongst the lowest in the economies with the highest overall hyper-integrated labour productivity growth.
4. The study by Garbellini and Wirkierman (2023), offering a conceptual, analytical, and empirical reconstruction of the debate between Solow (1957) and Pasinetti (1959) on productivity measurement.

The idea of using the subsystem (or vertically integrated sector) as a unit of analysis for the study of technical change has often been criticised (see, e.g., Schefold, 1982, p. 549, and Hagemann, 1987, p. 346). I believe this is mainly due to two misunderstandings.

First, vertical integration is sometimes considered going backwards in time, in a sort of 'neo-Austrian' perspective. I think the origin of this confusion comes from placing the *general* analytical device of vertical integration in the context of a *specific* joint production model (the 'pure fixed-capital system'), in which machines of different years are consolidated – through a discounting procedure applied to the price equations – in order to obtain a single-product system:

> We may call this operation 'vertical integration in a temporal sense' of the activities employing the machine in its various years of age; it allows us to formally eliminate the joint-production component and bring the analysis back to the forms of single production with only circulating capital (Baldone, 1980, p. 96).

Second, it is sometimes maintained that rates of productivity growth at the vertically integrated level are exogenous data of the analysis:

> The industry-specific nature of technical change also implies that, contrary to Pasinetti's assumption, rates of productivity growth in the different vertically integrated sectors cannot be thought of as being independent of each other (Hagemann, 1987, p. 346).

In this case, I think the origin of the confusion comes from taking a *specific* description of the technique in use present in Pasinetti's (1981) model – in which capital goods are produced by labour alone – and assigning to it a *general* interpretation of vertically integrated magnitudes.[42]

Pasinetti's (1981) 'intermediate case' (i.e., a description of technology without basic commodities) was thought of as a purely expository theoretical device. *In this particular case*, the rates of growth of direct labour coefficients coincide with those of vertically (hyper-) integrated labour coefficients. But this is clearly far from being a general principle.

In fact, it is apparent that vertically (hyper-) integrated magnitudes are derived from industry level ones, claiming no logical primacy or independence.[43] In any general empirical application, vertical integration is a device applied to existing direct (industry) magnitudes to obtain derived (vertically integrated) productivity growth rates.

This second source of misunderstanding warns, however, of the importance of making a clear statement as regards the aims and purposes of productivity analysis from the perspective of subsystems. As stated by Pasinetti himself:

> the analytical device of vertical integration *is not meant* to catch the detailed and localized sources of technical change; on the contrary, it is meant to synthesize the overall *effect* of technical change (whatever its sources, or nature, or remote localization in the economic system) on the final stage of production, of prices and of employment (Pasinetti, 1990, p. 258, emphasis in the original).

Thus, the focus is on measuring the *effects* of technical change on proportions, prices, and employment and not to study the *causes* behind changing productivities.

5.3. Measuring productive capacity

In a multisectoral economy, a scalar or subsystem measure of physical input per unit of output is not straightforward to obtain, given the multitude of output-input productivity ratios present in the economy. It is necessary to solve the aggregation of commodities, *or* the reduction of some of them in terms of others.

This is particularly difficult for capital goods, which, by being reproducible, are themselves subject to productivity improvements. In fact, in the summary record of the debate at the 1958 Conference on the Theory of Capital, Kaldor noticed two radically different positions on this issue:

> *One extreme case* was to assume that there was no technical progress in the production of capital goods but that these always required the same amount of real resources. This was obviously quite unrealistic. *At the other extreme*, one could say that a unit of capital was whatever unit was capable of producing a given output in a given year – ignoring both longer and shorter output streams. Here any distinction between the quantity of capital and its productivity was washed away by the definition itself. Any idea that capital might have varying productivities was lost; its output was always constant (Hague, 1961, p. 304, emphasis added).

The first 'extreme case' corresponds to the traditional TFP treatment of the 'quantity of capital', in which a TFP growth measure is assumed to capture disembodied efficiency changes, independently from capital deepening, which are assumed to require the same amount of *real* resources (e.g., 'waiting') per unit of saving.

The second 'extreme case' consists in measuring capital goods in 'units of capacity', i.e., as a set of composite commodities of heterogeneous physical content, specific for each final product

[42] See Garbellini (2010, pp. 152-155) for a development of the argument and Garbellini (2010, pp. 138-164) for a reply to other criticisms of the vertically (hyper-) integrated approach.
[43] See the early comments by Pasinetti (1963, Chapter V).

of the economy. But then, if capacity is defined in terms of the final output *actually* produced, at every moment the number of units of commodity-specific capacity would coincide with the number of units of each final product (Kaldor's idea of a 'constant output coming from capital').

However, by adopting such a measuring rod, the 'quantity of capital' in real terms would not be needed anymore, and, at the same time, each of these composite commodities would change their physical composition from one period to the next, though retaining their function as commodity-specific 'productive capacities'. This route was precisely the one taken by Pasinetti (1959) throughout his approach to structural economic dynamics.[44] What Kaldor observes is true, 'the difference between the quantity of capital and its productivity' is dispensed with. But this is not a problem when productivity measurement is *not* conceived from the value added side, as there is no net income to distribute among 'factors'. In fact, the procedure of using a *reduction* (through vertical integration) rather than an *aggregation* of capital goods is perfectly in line with adopting a subsystem perspective.

6. By way of conclusion

It is of course true that "to measure is not to understand" (Salter, 1966, p. 1), and this is particularly so as regards productivity analysis:

> One of the reasons why interpretative analysis of productivity has been slow to develop has been the interminable controversy over what is productivity and what do we really wish to measure. The word now carries a multitude of meanings; to some it measures the personal efficiency of labour; to others, it is the output derived from a composite bundle of resources; to the more philosophic, it is almost synonymous with welfare; and in one extreme case it has been identified with time. I personally believe that much of this discussion has proved fruitless and *only served to confuse the issues of measurement with the issues of interpretation*. Unless there is a revolution in statistical techniques and information, *only one type of productivity concept is measurable*. This is the concept of output per unit of input (Salter, 1966, p. 2, emphasis added).

Far from being a trivial statement, the position taken by Salter (1966) was not the usual one at the time of writing (and, clearly, even less nowadays). But it hopefully serves to clarify the broad overview presented in the preceding sections. In sum, I believe that applied productivity (and profitability) analysis *from a classical perspective* ought to be carried out by measuring changes in physical input use per unit of output (as seen from the expenditure side) using (growing) subsystems, as well as studying the effects that changes in physical productivity have on the (price) surplus of the system (as seen from the value added side), using production price systems.

Expenditure depends on commodity circulation, on material balances, on proportions: gross to net product. In sum, on physical outputs. Value added depends on commodity circulation-*cum*-exchange, and exchange implies relative prices (and so standards of value).

Productiveness starts from the exchange of commodities within the capitalist mode of production: net income and distributive rules must be faced. Productiveness is micro-economic; it can be applied to a single agent, a firm, an industry. And, to get to the overall economy, aggregation is necessary.

Productivity, instead, starts from the 'collective labourer', from the labour content in the circulation of commodities, revealing a continuous and dynamic process of division of labour, specialisation, automation and technological unemployment (labour saving trends), counteracted by effective demand. Productivity is macro-economic; there is no sense in searching for the

[44] See Pasinetti (1963, 1973, 1981, 1986, 1988). See also Garbellini and Wirkierman (2023).

productivity of an individual, as the crucial point for productivity analysis is general interdependence. That is why productivity measures start from the subsystem, the minimal unit of analysis, no matter the level of aggregation chosen. In fact, Pasinetti's (1959) first attempt at the concept of productivity was limited to an economy with two subsystems, but the two crucial ones: a sector producing the means of production, the other producing final uses. His intuitions could later be generalised, thanks to reduction and not aggregation.

Reduction, or expressing some commodities in terms of others, has turned out to be one important analytical device for productivity measurement from a classical perspective. Another one has been to measure capital goods in 'capacity units', i.e., in terms of the final commodities that can be reproduced with them. But then it is clear that, to have a capacity-unit of measurement, it becomes necessary to separate what re-enters the production process from what is a truly final use. And for this task it is the expenditure side, the physical surplus, which needs to be analysed. Its crucial component is investment: a *dual* concept in itself. Investment is a source of demand, of expenditure, but it also generates capacity, alters the means of production that have to be *priced* in the value added side.

Gross investment, i.e., (fixed) capital accumulation, plays a crucial role in productivity analysis, so the mechanism behind its treatment for productivity measurement in the context of empirical growing subsystems should be rendered explicit:

> in each year, the gross investment undertaken by each industry represents the flow of capital goods required to maintain the industry on its current growth path (Peterson, 1979, p. 220).

In fact, faced with multi-period accounting identities from the side of physical outputs, we observe only quantities produced. The separation between methods of production and activity levels of industries is analytical. If we assume unitary operation intensities, this leads to technique-*cum*-intensity level flow matrices. This observation means that it is not possible to separate, on entirely 'objective' grounds, growth from technical change in empirically given structures: empirical matrices contain growth rates.

All in all, on productivity analysis from a classical perspective, almost everything awaits to be empirically explored. This paper has been only a structured invitation to think about foundational concepts and some existing literature that may help it grow further.

In this sense, there are several directions for further research. With the consolidation of inter-country production networks and the availability of global inter-regional input-output (IRIO) data, it has become possible to measure productivity by representing a global value chain as an international subsystem producing a final product. But, while productivity became an international concept, competitiveness remained a national one, so it would be interesting to understand their interplay in a global economy. As regards technological change, new (conceptual and empirical) challenges emerge to measure productivity and profitability in an era of fast automation and industrial robotisation, where fixed-capital becomes ever-increasingly malleable (Wirkierman, 2022b, p. 274). If automation was taken to an extreme, would the indispensable role of labour in production be at risk? Finally, with the increasing digitalisation of production processes, there is a need to understand the nuances of productivity measurement when the differences between physical and digital outputs (and their associated inputs) become apparent (Wirkierman, 2022b, p. 280).

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