1. Introduction

Downs' influential result of convergence of parties' political platforms to the views of the median voter is achieved under the assumptions of opportunistic (purely office-motivated) parties and fixed voters' preferences (Downs, 1957). Since his seminal contribution, the literature on voting has investigated the conditions for platform convergence or divergence when one or more of the Downs' hypotheses are relaxed (see Grofman, 2004, for an overview). Relevant empirical literature shows that over the last twenty years, the distribution of voters and parties' agendas on specific issues have moved towards relatively extreme positions.[1] Several European countries, from France to Italy, from Germany to Hungary, have experienced a raise in the electoral support of far-right groups whose ideas used to be at the margins of the political discourse.[2] The possible drivers of this shift have been widely studied, with a number of works pointing to economic and distributional issues as possible causes.[3] From a theoretical perspective, dynamic election models with evolving state variables appear to be better suited for the analysis of a changing political landscape, in particular when economic variables are involved (Duggan and Martinelli, 2017).

This paper shows that if voters' preferences are influenced by factors identified by the empirical literature and evolve endogenously over time, the convergence of political platforms postulated by the median voter theorem is a special case in a range of possible outcomes that include political cycles and convergence to extreme political platforms.

We develop a behavioural dynamic model with heterogeneous individuals with endogenously evolving preferences and two policy-oriented parties. The parties have different core values in terms of income redistribution but the extent to which their policies are actually implemented depends on the relative support that they enjoy. Their policies change the income distribution and generate a feedback effect on electoral preferences. The binary political preferences of each individual are modelled using a discrete choice framework (McFadden, 2001; Train, 2009), in which the different factors that influence political choices have been treated in the relevant empirical literature[4]. More specifically, we include four different factors: (i) economic performance of the ruling party, (ii) income inequality, (iii) bandwagon effect, and (iv) heterogeneous individual factors. The first three factors are observable, while the fourth one is not.

The relationship between economic results and voting has been investigated by the literature on *responsibility hypothesis* (Lewis-Beck and Stegmaier, 2000; Lewis-Beck and

[1] See Boxell et al. (2020); Fiorina and Abrams (2008); Funke et al. (2016); Hobolt and Tilley (2016); McCarty et al. (2006), among many others.

[2] See Golder (2016); Halikiopoulou and Vlandas (2020); Lazaridis et al. (2016).

[3] For example, see Duca and Saving (2016); Kelly and Enns (2010); Garand (2010); Han (2016); McCarthy et al. (2006); Pontusson and Rueda (2010); Winkler (2019).

[4] For early evidence of time-evolving political preferences, see Fiorina (1981), Kiewiet (1983), and Kramer (1971).

Paldam, 2000): since voters hold the ruling parties accountable for the economic performance of a country, economic growth increases the possibility of their re-election. This causal nexus is also known as *economic voting* and has produced a large body of theoretical and empirical studies that have analysed the dependence of electoral choices on the main macroeconomic indicators (for a recent survey, see Lewis-Beck and Stegmaier, 2019).

The influence of *income inequality* in political preferences has roots in the political economy models along the lines of Alesina and Rodrik (1994) and Meltzer and Richard (1981). The level of income inequality can affect public preferences in two ways: (i) high level of inequality leads a larger share of voters to demand for a redistribution in order to improve their individual welfare (Aalberg, 2003; Finseraas, 2009; Piketty, 2018); (ii) due to a *hysteresis* effect, relatively high levels of inequality over time make inequality itself more acceptable for the public (Andersen and Yaish, 2012; Kelly and Enns, 2010).

The *bandwagon effect* was introduced by Leibenstein (1950) as a force that leads people "to wear, buy, do, consume, and behave like their fellows; the desire to join the crowd" (p. 184). This type of behaviour was proposed in order to explain "irrational" demand for certain goods. Simon (1954) independently formulated the same idea in relation to voting behaviour, conjecturing that people are more likely to vote for a candidate that is considered as the most likely winner. Recent empirical evidence have found support for bandwagon behaviour in voting (Morton et al., 2015).

Finally, the assumed heterogeneity in individual political preferences is related to voters' *partisan biases* (Brennan and Buchanan, 1984; Robbett and Matthews, 2018; Shayo and Harel, 2012). These biases are ideological priors that can lead voters to make choices that are independent from the expected election outcomes and, possibly, against their individual interests. As a consequence, despite the fact that an individual may benefit from one party's redistributive policies, she may still vote for the opposite party due to ideological and other non-economic reasons related to culture, religious beliefs, etc...

Two main results of this paper are worth mentioning. First, political polarization or convergence are mostly determined by the bandwagon effect and the responsibility hypothesis. The median voter theorem's result of convergence of policies appears to be a special case, which is achieved when the relative influences of the two behavioural factors are low. More precisely, convergence is achieved when the bandwagon effect is below a threshold that depends on the economic performance of the ruling party. Negative economic growth can outweigh the bandwagon effect and lead to a change in majority. Second, when voters prefer extremely high or low levels of inequality, large political swings and a higher level of polarisation are more likely.

In re-examinig the median voter's results, our framework connects and contributes to different literatures that have been developing in a mutually independent manner up to now. It brings together insights from dynamic elections with endogenous state variables (Azzimonti, 2011; Battaglini and Coate, 2007, 2008; Battaglini, 2014; Duggan and Forand, 2013) and responsible parties (Bernhardt et al., 2009; Calvert, 1985; Wittman, 1983), integrating empirical findings on the different factors affecting political preferences.

Our paper is closely related to Esponda and Pouzo (2019), who analyse the effects of bounded rationality and focus on how parties' previous performance (*retrospective voting*) can lead to polarisation in a static environment with policy motivated parties. Our work differs by including the bandwagon effects as a behavioral influence together with the parties' performance (in economic terms) and by adopting a dynamic setup. Looking at the results, while in both papers polarisation is the outcome of behavioural factors, in ours it can take the form of either shifts between the two extreme political positions or stability of one of them.

The paper also provides a methodological contribution, being the first attempt to analyse dynamic voting by means of a discrete choice framework with heterogeneous interacting agents. Discrete choice models in the same vein have been extensively used in financial economics, starting with the works of Brock and Hommes (1997, 1998) and Lux (1995) further developed by Chiarella and He (2001); Chiarella et al. (2006); Westerhoff and Dieci (2006); Dieci and Westerhoff (2016) among many others, in macroeconomics (for example De Grauwe, 2012; Flaschel et al., 2018; Frankel and Froot, 1987, 1990), and, more recently, in epidemiology to incorporate behavioural factors (Baskozos et al., 2020).

The remainder of the paper is structured as follows. Section 2 presents the model's assumption for the baseline version of our framework while section 3 illustrates the analytical and numerical results. Section 4 introduces and examines two extensions of the baseline model. The results are discussed in section 5 while section 6 offers some concluding remarks.

2. The Model

The model is composed of two political parties and a large number of heterogeneous voters with evolving preferences. Voting behaviour depends on the four factors listed above: responsibility hypothesis, income inequality across voters, bandwagon effect, and individual biases. The two parties have different core values but the extent to which they pursue them depends on their relative support.

2.1. Left and Right

We assume an economy composed by $2N$ boundedly rational voters, with N very large. Each voter has the option to choose between two different political parties, and call these left (denoted by L) and right (denoted by R). The parties differ in their views on redistribution: L favours top-down redistribution in order to reduce inequality, which is quantified by the Gini coefficient for income g, while R, in contrast, aims to increase

g through a redistribution from bottom to top income earners.[5] Both parties are policy oriented, in the sense that they only support the redistribution policy that is consistent with their platform, even when it is not the one desired by the majority of voters.

Let the x be the relative support of the left, such that

$$x = \frac{n^L - n^R}{2N}, \tag{1}$$

where n^L is the share of the left voters and n^R the share of voters who support the right, such that $n^L + n^R = 2N$. This implies that $x \in [-1, 1]$, with $x > 0$ when $n^L > n^R$.

2.2. Political Choices

We assume that all individuals' preferences are *exhaustive* such that if an individual does not choose one party, they necessarily choose the other. Along the lines of discrete choice models, individual preferences depend on observable characteristics, which are common across agents, and unobservable idiosyncratic ones. The utility for individual i of choosing L is given by

$$U_i = \beta \mathbf{v} + \epsilon_i, \tag{2}$$

where $\mathbf{v}$ is a column vector of the observable factors, β a row vector which captures the relative importance of each of the elements of v, and ϵ_i represents the unobservable characteristics for voter i. Since the utility function (2) can be positive or negative, the political choice $\mathcal{C}_i = \{L, R\}$ for i can be expressed as

$$\mathcal{C}_i = \begin{cases} L \ \text{ if } \beta \mathbf{v} + \epsilon_i > 0, \\ R \ \text{ if } \beta \mathbf{v} + \epsilon_i \leq 0, \end{cases} \tag{3}$$

The vector $\mathbf{v}$ includes three types of factors: the *bandwagon effect*, the *responsibility hypothesis*, and *inequality*, and $\beta = [\beta_x, \ \beta_y, \ \beta_q]$, where $\beta_x, \ \beta_y, \ \beta_q$ quantify the relative importance of the three effects, respectively. For simplicity, $\beta_x, \ \beta_y, \ \beta_q$ are all positive and, without loss of generality, we set $\beta_q = 1$. Accordingly, the three effects can be modelled as follows:

E1. Bandwagon effect: from (3), given that $\beta_x > 0$ and x expresses the relative proportion of left voters, the bandwagon effect is simply expressed by x.

E2. Responsibility hypothesis: macroeconomic performance is quantified by the change in output y denoted by[6] $\dot{y}$. Since $\dot{y} > 0$ (< 0) is expected to have a positive

[5] We are aware that associating left and right to a top-down and down-top redistribution, respectively, might not accurately represent the current ideologies in two-party systems. The denomination of "left" and "right" is adopted here as a generic reference for the reader's convenience.

[6] In general, for any variable $z \in \mathbb{R}$ let $\dot{z}$ denote its time derivative.

(negative) effect for the ruling party, and given that $\beta_y > 0$ in (3), the responsibility hypothesis effect can be expressed by $x\dot{y}$.

E3. Inequality: as argued by Alesina and Angeletos (2005), in each society it is possible to quanfity a level of inequality that is socially acceptable due to differentials in effort. Let us identify this level with g_0. Accordingly, as inequality becomes higher (lower) than this level, people on average will favour a redistribution towards the bottom (top) income earners.[7] Hence, on average, voters choose L if $g > g_0$ and R if $g < g_0$.

Considering E1-E3, the vector $\mathbf{v}$ can be written as

$$\mathbf{v} = \begin{bmatrix} x \\ x\dot{y} \\ g - g_0 \end{bmatrix}, \tag{4}$$

which implies

$$\beta\mathbf{v} = \beta_x x + \beta_y x\dot{y} + g - g_0. \tag{5}$$

We assume that ϵ_i follows a logistic distribution.[8] Accordingly, the probability $P(L|\mathbf{v})$ that a randomly chosen individual chooses L, for a given $\mathbf{v}$ can be expressed as

$$P(L|\mathbf{v}) = \frac{e^{\beta\mathbf{v}}}{1 + e^{\beta\mathbf{v}}}. \tag{6}$$

Accordingly, the probability for given $\mathbf{v}$ of choosing R is calculated as

$$P(R|\mathbf{v}) = 1 - P(L|\mathbf{v}) = \frac{1}{1 + e^{\beta\mathbf{v}}}. \tag{7}$$

From (1), we can write $n^L = (1 + x)N$ and $n^R = (1 - x)N$. Then from (6) and (7),

[7]Since g_0 can be considered as the preference of the median voter, this modelling choice also accounts for micro-level factors. Modifications in income distribution can change the position of single voters within the distribution and, as a consequence, individual policy preferences. For example, a voter is expected to oppose a redistribution from top to bottom that makes her worse off (and therefore she favours the political right). However, if the distribution of income becomes more concentrated, the same voter might find herself now benefiting from the same type of redistribution and, accordingly, she becomes more likely to vote left.

[8]The assumption of a logistic distribution implies that the discrete choice process follows a logit model, as common in empirical research (Train, 2009). The logistic distribution is also the standard implicit assumption in the theoretical models drawing on discrete choice theory (see the survey by Hommes, 2006, for example).

the change in the relative difference is given by

$$\dot{x} = (1-x)\frac{e^{\beta \mathbf{v}}}{1+e^{\beta \mathbf{v}}} - (1+x)\frac{1}{1+e^{\beta \mathbf{v}}}. \tag{8}$$

From (8), it can be seen that x plays both a positive and a negative role in its own evolution. This double effect appears more clearly if we re-express (8) as follows:

$$\dot{x} = \frac{e^{\beta \mathbf{v}} - 1}{1+e^{\beta \mathbf{v}}} - x, \tag{9}$$

where the derivative of the first component on the right hand side with respect to x is positive as $\beta \mathbf{v}$ is increasing in x and $\frac{e^{\beta \mathbf{v}}-1}{1+e^{S}}$ is increasing in $\beta \mathbf{v}$.

The economic intuition behind equation (9) is that, on the one hand, when the relative population supporting the left grows (shrinks), the probability of switching to the left increases (decreases) while the probability of switching to the right decreases (increases). On the other hand, the consequent increase (decrease) in x implies that the probability of switching to the left influences a smaller (greater) share of voters as it only applies to the share of the right (left). From (9), the direct negative effect of $x > 0$ to the $\dot{x}$ is linear while the indirect one is increasing and concave. Hence, we expect that (depending on the parameter values) for $x > 0$ after a certain level of x the overall effect on $\dot{x}$ will be negative. Similarly for $x < 0$ below a certain level of x the positive effect on $\dot{x}$ will dominate the negative one.

2.3. Redistribution

In a responsive democracy, the public perception has a feedback effect on inequality and economic performance through the choices of the elected officials, who will shape their policies according to the public's taste in the attempt of being re-elected (Wlezien, 2004). With specific reference to redistribution, Brooks and Manza (2006) find evidence of social preferences influencing social policy output, while Cusack et al. (2006) provide support for the hypothesis that rising inequality increases the demand for redistribution.

As discussed earlier, when in power the left (right) party decreases (increases) inequality to an extent proportional to its political support. As a consequence, we can write

$$\frac{\partial \dot{g}}{\partial x} < 0. \tag{10}$$

In order for $g \in [0;1]$, the following conditions should hold:

$$g \leq 1 \Leftrightarrow \dot{g} \leq 1 - g, \tag{11}$$

$$g \geq 0 \Leftrightarrow \dot{g} \geq -g. \tag{12}$$

Furthermore, it is reasonable to assume that as g gets closer to 1 (0), it becomes progressively more difficult to raise (lower) it further. This implies that $\dot{g}$ should be convex for low values of g and concave for high values of g. On the base of these considerations, we can express $\dot{g}$ as

$$\dot{g} = -xg(1-g). \tag{13}$$

In conclusion, the model dynamics is described by the changes in three variables: x, g, and y. For our analytical study, we will consider output growth as exogenous. This choice allows us to obtain some general results, focusing on the bandwagon effect.[9] In the next section, we analyse the properties of the following dynamical system

$$\dot{x} = (1-x)\frac{e^{\beta\mathbf{v}}}{1+e^{\beta\mathbf{v}}} - (1+x)\frac{1}{1+e^{\beta\mathbf{v}}} \tag{8}$$

$$\dot{g} = -xg(1-g) \tag{13}$$

3. Results

Definition 1. *Let $\boldsymbol{z} = (z_1, z_2, ..., z_n)$. For any dynamical system $\dot{\boldsymbol{z}} = f(\boldsymbol{z})$, a stationary equilibrium is defined as the state in which $\dot{\boldsymbol{z}} = 0$.*

The stationary equilibrium we refer to is a statistical one, in the sense that stationarity at the aggregate level does not necessarily imply no variations at the micro level: individuals can still be changing their political preferences while $\dot{x} = 0$.

Considering $\beta_y \dot{y}$ as exogenously given, with output growth as a constant, we can set $\beta_y \dot{y} = \sigma$, with $\sigma > (<)0$ indicating positive (negative) growth. Let $\mu = \beta_x + \sigma$ capture the aggregate effect of bandwagon and growth. Accordingly, equation (5) becomes

$$\beta\mathbf{v} = \mu x + g - g_0 \tag{14}$$

Considering one period approximately equal to a quarter and being σ a constant, for the stability analysis we can consider $\mu > 0$, abstracting from unrealistically large negative constant growth. For completeness, the case of $-\sigma > \beta_x \Rightarrow \mu < 0$ is investigated numerically at the end of the section.

3.1. Stability and convergence

Proposition 1. *Consider the system of equations (8), (13), and (14). For $\mu \leq 2$,*

(i) there exist exactly three stationary equilibria: $\{0, g_0\}$, $\{x_0^1, 0\}$, with $x_0^1 \in (-1, 0)$ and $\{x_1^1, 1\}$, with $x_1^1 \in (0, 1)$.

[9] In this simplified setting, it is still possible to assess the role of economics growth with respect to the bandwagon effect. As shown below, the analysis of the stability defines a relationship between the bandwagon effect and the effect of income growth.

(ii) $\{0, g_0\}$ *is locally stable if* $\mu < 2$ *and a centre for* $\mu = 2$,

(iii) both $\{x_0^1, 0\}$ *and* $\{x_1^1, 1\}$ *are saddle points.*

Proof

See Appendix.

The first part of Proposition 1 states that for $\mu \leq 2$ there exist three stationary equilibria: one, defined as *centrist*, in which the voters are equally split between the two parties and inequality is at the average socially acceptable level; one with a right-wing majority and zero inequality; and one with a left-wing majority and maximum inequality. The second and third parts of Proposition 1 imply that, as long as the combined effect of the bandwagon and the responsibility hypothesis are relatively low, the centrist equilibrium is locally (asymptotically) stable.

As the key parameter μ depends on the joint effect of the bandwagon effect and economic growth, when the economy is growing ($\sigma > 0$), the economic performance compounds the bandwagon effect, while negative growth reduces it, by creating discontent for the management of the economy by the party in charge.

For the special case of $\mu = 2$, Proposition 1 states that the relative population of voters oscillates around the centrist equilibrium.
The parameter g_0 is assumed to be constant, at least in the short run, since it represents a structural factor in a society, which is unlikely to change as sharply as political preferences. The effects of an endogenously evolving g_0 are numerically analysed in section 4.

Corollary 1. *In the system of equations (8), and (13), the* $\{0, g_0\}$ *stationary equilibrium, has two complex conjugate eigenvalues when* $2 - 2\sqrt{2(g_0 - g_0^2)} < \mu < 2 + 2\sqrt{2(g_0 - g_0^2)}$.

Proof

See Appendix.

Corollary 1 is complementary to Proposition 1, showing that the local stability of the centrist equilibrium takes the form of a spiral sink. Hence for $\mu > 0$, even in the case of political stability of the centre, some level of (mild) political volatility exists. Note that the further away g_0 is from 0.5 (in either direction), the smaller will be the interval for μ for which the equilibrium has complex eigenvalues, which means that whether the stability of the centrist equilibrium is cyclical or not also depends on the level of socially accepted inequality.

Figure 1 shows that, starting from $g = g_0$ and $x \neq 0$, the system is unstable. Starting with a slight left (right) majority the voters soon shifts to the right (left) as inequality falls below (raises above) the acceptable level. When inequality reaches the upper (lower) limit, the driving force of the bandwagon effect is exhausted and voters begin to shift to

the left (right). Since when $g = g_0$, we always have that $x \neq 0$, the stationary equilibrium is never achieved. Changing the parameter setting such that $\mu < 2$, the model displays cyclical convergence to $\{0, g_0\}$, as shown by figure 2 in which the initial condition is set as $g = 0.4$. To complete the analysis, figure 3 shows the cyclical behaviour of the economy when $\mu = 2$.
[Figure 1 here]
[Figure 2 here]
[Figure 3 here]

Figures 4 and 5 present phase diagrams to illustrate the cyclical convergence to the $\{0, g_0\}$ equilibrium, while figures 6 and 7 present the convergent behaviour that occurs for $\mu = 2$.
[Figures 4 and 5 here]
[Figures 6 and 7 here]

3.2. From convergence to the extremes

Proposition 2. *Consider the system of equations (8) and (13) and let $\mu > 2$. There exist $\bar{\mu}(g_0) > 2$ and $\hat{\mu}(g_0) > 2$, such that, apart from the $\{0, g_0\}$, $\{x_0^1, 0\}$ and $\{x_1^1, 1\}$, the following stationary also equilibria exist:*

(i) $\{x_0^3, 0\}$, *with* $x_0^3 \in (0, 1)$, *if* $\mu = \bar{\mu}(g_0)$

(ii) $\{x_0^2, 0\}$, *with* $x_0^2 \in (0, 1)$ *and* $\{x_0^3, 0\}$, *with* $x_0^3 \in (0, 1)$, *if* $\mu > \bar{\mu}(g_0)$,

(iii) $\{x_1^3, 1\}$, *with* $x_0^3 \in (-1, 0)$, *if* $\mu = \hat{\mu}(g_0)$,

(iv) $\{x_1^2, 1\}$ *with* $x_1^2 \in (-1, 0)$, *and* $\{x_1^3, 0\}$, *with* $x_1^3 \in (-1, 0)$, *if* $\mu > \hat{\mu}(g_0)$.

Proof

See Appendix.

Proposition 2, shows that for $\mu > 2$, the number of equilibria depends indirectly on g_0 and whether $\bar{\mu}(g_0) \lesseqgtr \hat{\mu}(g_0)$. Based on the previous intuition, we expect that the dynamics of the economy will depend on the importance of the bandwagon effect and whether $\bar{\mu}(g_0) \lesseqgtr \hat{\mu}(g_0)$, as this leads to the existence of new politically extreme equilibria.

Proposition 3. *The following are true:*

(i) $\frac{\partial \bar{\mu}(g_0)}{\partial g_0} > 0$ *and* $\frac{\partial \hat{\mu}(g_0)}{\partial g_0} < 0$,

(ii) *if* $g_0 = 0.5$ *then* $\bar{\mu}(g_0) = \hat{\mu}(g_0)$.

Proof

See Appendix.

Proposition 3, says that the higher the value of g_0, the higher (lower) will be the level of μ sufficient for the existence of equilibria with a left (right) majority and zero (complete) inequality. Also from Proposition 3, it follows that:

Remark 1. *If $g_0 < 0.5$ then $\bar{\mu}(g_0) > \hat{\mu}(g_0)$, while if $g_0 > 0.5$ then $\bar{\mu}(g_0) < \hat{\mu}(g_0)$.*

This result highlights the importance of the socially accepted level of inequality in the existence of the various political equilibria. If, for example, $g_0 > 0.5$ (high level of socially accepted inequality) and $\bar{\mu}(g_0) < \mu < \hat{\mu}(g_0)$, then we expect the economy to move towards the right equilibrium, which will be stable.

Figures 8 and 9 present the limit cycle that emerges when $2 < \mu < \min\{\bar{\mu}(g_0), \hat{\mu}(g_0)\}$.

[Figures 8 and 9 here]

Figures 10 and 11 present the cases $g_0 > 0.5$ and $\mu \in (\hat{\mu}(g_0), \bar{\mu}(g_0),)$ and $g_0 < 0.5$ and $\mu \in (\bar{\mu}(g_0), \hat{\mu}(g_0))$, respectively.

[Figure 10 here]

[Figure 11 here]

In this case, besides the three unstable equilibria, there exist two more equilibria: $\{x_1^2, 1\}$ with $x_1^2 \in (-1, 0)$, and $\{x_1^3, 1\}$, with $x_1^3 \in (-1, 0)$. The system is expected to converge to $\{x_1^3, 1\}$ because it implies total inequality in income distribution and right-wing majority.

Finally, figures 12 and 13 illustrate the dynamics for $\mu > \max\{\bar{\mu}(g_0), \hat{\mu}(g_0)\}$. [Figures 12 and 13 here]

In this case there will exist seven equilibria in total: $\{0, g_0\}$; $\{x_0^1, 0\}$, with $x_0^1 \in (-1, 0)$; $\{x_0^2, 0\}$, with $x_0^2 \in (0, 1)$; $\{x_0^3, 0\}$, with $x_0^3 \in (0, 1)$; $\{x_1^1, 1\}$, with $x_1^1 \in (0, 1)$; $\{x_1^2, 1\}$ with $x_1^2 \in (-1, 0)$; and $\{x_1^3, 1\}$, with $x_1^3 \in (-1, 0)$. Both $\{x_0^3, 0\}$ and $\{x_1^3, 1\}$ are expected to be stable: the former implies perfect equality with left-wing majority and in the latter the majority is right-wing and inequality at its maximum. For the reader's convenience, the different equilibria are summarised in table 1.

[Table 1 here]

Figures 14 and 15 present the bifurcation diagrams for μ and g_0, respectively, including negative values for μ.

[Figure 14 here]

[Figure 15 here]

Figure 14 confirms the results of the stability analysis, showing that the system is stable for $0 < \mu < 2$ and the emergence of a limit cycle and extreme equilibria for

$\mu > 2$. Moreover, the bifurcation diagram shows that stability persists for relatively mild negative growth ($\mu > -2$). For $\mu < -2$, the electorate keeps swinging between the two parties because the extremely negative economic performance pushes voters to punish the majority party. In other words, large negative growth generates a reverse bandwagon effect: it is the minority party that is going to be preferred. The swings are larger the worse is the economic performance.

The bifurcation diagram in figure 15 summarises the voters' behaviour for different values of g_0, highlighting the role of the socially perceived level of inequality on political choices. If g_0 is close to 0.5, we observe fluctuations, but for values closer to 0 (1) a clear left (right) majority emerges.

4. Extensions

This section extends the baseline model to account for the feedback effect detected by the literature between income inequality and growth and between income distribution and the acceptable level of income inequality. As shown below, the main results of the model do not appear to change and for this reason the analysis is limited to numerical simulations.

4.1. Inequality and output

While the electoral behaviour is affected by economic performance according to the responsibility hypothesis as discussed, the level of inequality affects both growth and public's preferences. As for the former, the literature within the tradition of Downs (1957) argues that inequality slows down growth only indirectly, by inspiring policies that reduce the accumulation of capital (Alesina and Rodrik, 1994). However, Bertola et al. (2006) and Berg et al. (2018) found little empirical support for Alesina and Rodrik's thesis. A strand of recent literature proves that the relationship between income inequality and growth is more complex and multifaceted (Piketty, 2014; Cynamon and Fazzari, 2015, among others).

The presence of nonlinearities in the inequality-growth relationship is known since Kuznets (1955). Recently, Grigoli and Robles (2017) have empirically analysed the functional relationship between growth and the level of inequality measured by the Gini coefficient for a large sample of countries and estimated a polynomial relationship. Using their estimate for the functional relationship non-conditional to the control factors that they test, we model the relationship between income inequality and output as follows[10]

$$y = exp(\gamma_1 g^2 + \gamma_2 g + \gamma_3) \tag{15}$$

[10] Grigoli and Robles (2017) use net Gini coefficients and not market coefficients. Given the qualitative nature of the present analysis, we adopt their functional form and coefficient estimates since they provide an insightful extension of the model in presence of (empirically verified) nonlinearities in the growth-inequality relationship.

In the estimates by Grigoli and Robles (2017), equation (15) has a maximum at 0.12 for the non-conditional relationship and around 0.27 for the conditional one.

The parameters for the numerical simulations of the model are set as follows: $\beta_x = 2.5; \beta_y = 1.5; g_0 = 0.5; T = 100$. The values of the parameters in (15) are set as in Grigoli and Robles (2017): $\gamma_1 = -0.0001; \gamma_2 = 0.0024; \gamma_3 = .077$.

[Figure 16 here]

Figure 16 shows that, even taking into account the relationship between inequality and growth, the main results of the baseline model do not seem to be qualitatively affected, despite the presence of an economic cycle along with the political one. The model generates cycles with a duration of about 30 period. In both the baseline and the extended model, the length and the amplitude of the cycle depend on the relative influence of the bandwagon effect.

The expansionary phase in income is driven by a sharp decrease in inequality, which results from a rapid shift in public opinion leading to a left-wing majority. While inequality continues to decrease, income appears to reach a ceiling. The stagnant income decreases the support of the left and, as the Gini index approaches zero, a shift in public opinion determines a right-wing majority. The combination of negative growth, caused by higher income inequality, and the high levels of the Gini index will determine a change in political preferences that will restart the cycle.

4.2. Endogenous desirable inequality

As we have already mentioned, the level of inequality affects public preferences in two relevant and apparently idiosyncratic ways. First, high level of inequality leads voters to demand for a redistribution (Finseraas, 2009). This effect is discussed in the baseline version of the model introduced in section 2. Second, a *hysteresis* effect has been detected: historically high level of inequality makes inequality itself more acceptable for the public (Kelly and Enns, 2010).

Andersen and Yaish (2012), using survey data across twenty countries, find evidence about the influence of the social classes of the respondents and the historical levels of the Gini index on the public's perception of inequality. In particular the effect of the Gini coefficient for net income inequality on the acceptable level of inequality appears to be significant and sizeable even when controlling for different alternative explanatory variables. They use a linear regression together with a number of controls which cannot be reproduced in our model.

Since in our model the Gini index g is a function of x, we represent this effect by allowing g_0 to change over time depending on x. The relationship between the two variables is represented as a logistic, which captures the fact that variations at the extremes should be relatively smaller.

Furthermore, in order to reasonably limit the range of variation of the desirable level of inequality, the admissible values g_0 are restricted within the empirical minimum and maximum of the Gini index recorded in the US in the period 1967-2019, which are 0.386 and 0.489, respectively. The data are taken from the St Louis Fed and the minimum and maximum were recorded in 1968 and 2017, respectively. Accordingly, in this extension of the model we set

$$g_0 = \frac{0.88}{1 + exp(0.23x)} \tag{16}$$

where the two constants 0.88 and 0.23 are chosen such that $g_0 = 0.386$ when $x = 1$ and $g_0 = 0.489$ when $x = -1$, hence ensuring that $0.389 \leq g_0 \leq 0.489$.

Once again, the qualitative insights provided in the baseline model hold. As shown by figure 17, the patterns produced by the simulations of this extended model are similar to those of section 4.1. However, in this setting g_0 follows the dynamics of x, delaying the phase transition, slightly lengthening the duration of the cycle. In particular, g reaches almost 1 before the public opinion changes and redistribution occurs.

[Figure 17 here]

5. Discussion

According to our results, the convergence or polarisation of the political system depend on the interaction among bandwagon effect, economic growth, and public perception of inequality.

The bandwagon effect emerges as the main factor in determining the degree of convergence or divergence in the system, in particular when accompanied by economic growth. More precisely, it is possible to identify a critical level of strength of the bandwagon effect below which the system is led to a centrist equilibrium. Conversely, a relatively stronger effect can drive the system to extreme equilibria. In the case of stability of the centrist equilibrium, political convergence between the two parties predicted by the median voter theorem emerges through a different and original channel with respect to the standard treatment. Indeed here, political convergence towards the centre in terms of redistributive policies is not achieved because parties, in the attempt of maximising their chances of being elected, choose a policy that is more likely to satisfy voters' preferences as in the standard Downsian framework. Here, the population moves towards being equally split between the two parties and as a consequence, redistribution converges to the average socially acceptable level because neither party has enough support for a redistribution.

The threshold for the bandwagon effect below which stability is achieved is not constant but depends on the interaction between the public perception of the economic growth and the level of accepted inequality. In particular, when aggregate income decreases and the public attach a strong importance to the economic performance, the strength of the bandwagon effect appears to vanish. In other words, economic growth amplifies the effect of the bandwagon effect in good times while an economic crisis is a possible source of voters' swings, instability, and polarisation.

Also the public perception of inequality plays a significant role. Specifically, the further is the level of accepted inequality from its central value of 0.5, the wider are the fluctuations in voters' preferences in the cyclical convergence to the centrist equilibrium. For values of g_0 close enough to one of the extremes, polarisation increases and extreme equilibria become more likely.

Despite our model being limited in scope and extremely parsimonious, its results can have some relevance in the current shifting political landscape. According to Schmitt-Beck (2015), the bandwagon effect is stronger in case of detachment of voters or with little available information about the candidates. While the detachment of voters is confirmed by the declining voting participation rates, the direction of the changes in the level and quality of information in the era of social media is not clear. If the larger use of social media increases the amount of information available for voters, as argued by Ernst et al. (2019), it can also directly strengthen the bandwagon effect. However, social media can also work as an echo chamber or vehicles for *fake news* (Törnberg, 2018) and as a consequence decrease the quality of information. Even in this alternative scenario, the use of social media can still increase the intensity of the bandwagon effect through two different channels. First, a selective collection of contacts and sources in social media can reduce the interaction between groups with opposite political persuasions, and consequently prevent voters from being exposed to the arguments of the opposite side. Second, the echo chambers, by strengthening individual beliefs, can lead voters towards more extreme views.

Our analysis can also provide some context for phenomena that are not immediately revealed by the two-party preference, as the emergence of relatively more extreme positions within each party (for the US see McCarty et al., 2006) or the voting for alternative or fringe parties (in the UK and other European countries). From this perspective, the model integrates the typical narrative of the median voter, by showing how a more polarised electorate in two-party system drives the parties towards more radical stances.

Finally, the findings of our paper point to the increase in political divergence as a result of the combination of low growth and increasing inequality, both of which have plagued some advanced economies in the last decades. Moreover, the model points to a higher tolerance of the public for inequality as one of the possible reasons for more extreme voting choices.

The qualitative behaviour of the model revealed by analytical results is substantially confirmed by the extensions that endogenise output and acceptable inequality.

6. Concluding remarks

This paper introduces a novel framework for the analysis of dynamic voting in a two-party system. Voters have dynamically evolving preferences that are affected by social, economic, and idiosyncratic factors, which in turns are determined by the economic policies of the governing party. The number and the type of equilibria crucially depend on the bandwagon effect, which pushes individuals to follow the majority.

If the influence of the bandwagon effect is relatively low, the population is equally split between the right and the left. This split determines a convergence in terms of the policies of the two parties, as postulated by the standard median voter framework. A relatively high influence of the bandwagon effect is a necessary and sufficient condition for the instability of the previous equilibrium and at the same time for the existence of equilibria which correspond to politically extreme situations with strong left or right political majorities. Political swings and a higher level of polarisation can also be the result of extreme voters' preferences in terms of acceptable inequality.

The paper adds to the literature from a twofold perspective. Firstly, it provides a novel framework for the modelling of dynamic voting and its integration with economic models, which features endogenously evolving state variables for voters and includes the consolidated results of the median voter theorem as a special case. Secondly, it proposes an original treatment for popular discrete choice models that introduces new perspectives for the study of the emergence of political convergence or polarisation as dependent on multiple factors.

Our parsimonious framework is flexible enough to be extended in a number of possible different directions. First, the analysis could be enriched by allowing for abstention as a third option. Indeed, Downs (1957) himself considered the possible implications of abstention, while Grofman (2004) argues that extreme voters are the most likely to abstain from voting. Our model is well equipped to study the incidence of lower turnout of extreme voters on convergence or divergence of parties' policies. Second, the framework can be also extended along similar lines through the inclusion of a third party, better representing continental Europe system and other western economies other than the UK or the US. Third, the range of factors affecting the political preferences could be enlarged to include, for example, unemployment or the functional distribution of income. This extension could encompass the analysis of Piketty (2018) of a partition of the electorate across social divides. A fourth possible direction concerns the inclusion of existing growth models in order to achieve a more refined representation of the relationship between inequality and growth. Finally the parties' behaviour can be enriched, allowing for the parties to have more than one political goals and to be opportunistic.

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Appendix

Proof of Proposition 1

Existence

In order to prove the existence of the first stationary equilibrium, we just need to set $x = 0$ and $g = g_0$ in (8) and (13), respectively. Regarding the existence of the other two stationary equilibria, we will analyse the cases of $g = 0$ and $g = 1$ separately.

(i) If $g = 0$ then $\dot{x} = 0$, if and only if

$$(1-x)\frac{e^S}{e^S + e^{-S}} = (1+x)\frac{e^{-S}}{e^S + e^{-S}}$$

or

$$e^{2[\mu x - g_0]} = \frac{1+x}{1-x} \tag{17}$$

Define the following real valued function $F : (-1, 1) \to \mathbb{R}$, with

$$F(x) = e^{2[\mu x - g_0]} - \frac{1+x}{1-x},$$

such that $F(x) = 0$ if and only if $\dot{x} = 0$. Thus, it is sufficient to show that there exists $x_0^1 \in (-1, 0)$ such that $F(x_0^1) = 0$. Note that

$$\lim_{x \to 1} F(x) = -\infty, \tag{18}$$

$$\lim_{x \to -1} F(x) > 0, \tag{19}$$

which means that given continuity there exists at least one $x_0^1 \in (-1, 1)$ such that $F(x^1) = 0$. The point will be unique if the derivative

$$F'(x) = -1 - \frac{e^{\mu x - g_0}(-1 + e^{\mu x - g_0})\mu}{(1 + e^{\mu x - g_0})^2} + \frac{e^{\mu x - g_0}\mu}{1 + e^{\mu x - g_0}},$$

is negative, hence $F(x)$ strictly decreasing. $F'(x)$, can be expressed alternatively as

$$F'(x) = -1 + 2\mu\frac{e^{\mu x + g_0}}{(e^{\mu x} + e^{g_0})^2}.$$

For $\mu < 2$,

$$F'(x) < -1 + 4\frac{e^{\mu x + g_0}}{(e^{\mu x} + e^{g_0})^2},$$

but given that

$$4\frac{e^{\mu x + g_0}}{(e^{\mu x} + e^{g_0})^2} < 1,$$

as

$$4e^{\mu x + g_0} - (e^{\mu x} + e^{g_0})^2 = (e^{\mu x} - e^{g_0})^2 > 0,$$

we get that

$$F'(x) < 0.$$

Hence, given continuity, there exists a single $x_0^1 \in (-1, 1)$ such that $F(x_0^1) = 0$. We also need to show that $x_0^1 \in (-1, 0)$. Note that, given that $F'(x) < 0$, in order to show that $x_0^1 \in (-1, 0)$, it is sufficient to show that $F(0) < 0$. Note that

$$F(0) = e^{-2g_0} - 1,$$

which is negative for all $g_0 > 0$.

(ii) If $g = 1$ then $\dot{x} = 0$, if and only if

$$e^{2[\mu x + 1 - g_0]} = \frac{1 + x}{1 - x} \tag{20}$$

Following the same steps as in the previous part, we can show that

$$\Phi(x) = e^{2[\mu x + 1 - g_0]} - \frac{1 + x}{1 - x},$$

is also strictly decreasing and given that $-g_0 + 1 > 0$, then $\Phi(x_1^1) = 0$ with $x_1^1 \in (0, 1)$.

Stability

For the first equilibrium, let us calculate the Jacobian matrix at the $\{0, g_0\}$ stationary equilibrium.

$$\frac{\partial \dot{x}}{\partial x} = -1 - \frac{e^{\mu x + g - g_0}(-1 + e^{\mu x + g - g_0})\mu}{(1 + e^{\mu x + g - g_0})^2} + \frac{e^{\mu x + g - g_0}\mu}{1 + e^{\mu x + g - g_0}}$$

Hence $J_{11} = -1 + \mu/2$.

$$\frac{\partial \dot{x}}{\partial g} = \frac{2e^{\mu x + g + g_0}}{(e^{\mu x + g_0} + e^{g_0})^2}$$

which gives $J_{21} = \frac{1}{2}$

$$\frac{\partial \dot{g}}{\partial x} = -g(1-g),$$

hence $J_{21} = g_0^2 - g_0$.

$$\frac{\partial \dot{g}}{\partial g} = -x(1-2g)$$

which means that $J_{22} = 0$.

Then the Jacobian matrix is,

$$J = \begin{bmatrix} -1+\mu/2 & 1/2 \\ g_0^2 - g_0 & 0 \end{bmatrix} \tag{21}$$

$$Tr(J) = -1 + \mu/2,$$
$$|J| = -(g_0^2 - g_0)/2 > 0,$$

Accordingly, for $\mu < 2$ the sum of the eigenvalues is negative and their product is positive, hence both eigenvalues are negative.

For $\mu = 2 \Rightarrow Tr(J) = 0$, confirming that the stationary equilibrium is a stable centre.

A necessary condition for the $\{x_0^1, 0\}$ equilibrium to be stable is the determinant of the Jacobian at that point to be positive.

Note that at $\{x_0^1, 0\}$,

$$J_{11} = -1 - \mu \left[\frac{e^{\mu x_0^1 - g_0}(-1 + e^{\mu x_0^1 - g_0})}{(1 + e^{\mu x_0^1 - g_0})^2} - \frac{e^{\mu x_0^1 - g_0}}{1 + e^{\mu x_0^1 - g_0}} \right],$$

or,

$$J_{11} = -1 - \mu e^{\mu x_0^1 - g_0} \left[\frac{-1 + e^{\mu x_0^1 - g_0}}{(1 + e^{\mu x_0^1 - g_0})^2} - \frac{1}{1 + e^{\mu x_0^1 - g_0}} \right]$$

or,

$$J_{11} = -1 - \mu e^{\mu x_0^1 - g_0} \left(\frac{-1 + e^{\mu x_0^1 - g_0}}{(1 + e^{\mu x_0^1 - g_0})^2} - \frac{1 + e^{\mu x_0^1 - g_0}}{(1 + e^{\mu x_0^1 - g_0})^2} \right)$$

or,

$$J_{11} = -1 + \frac{2\mu e^{\mu x_0^1 - g_0}}{(1 + e^{\mu x_0^1 - g_0})^2}$$

$$J_{12} = \frac{2e^{\mu x_0^1 + g_0}}{(e^{\mu x_0^1 + g_0} + e^{g_0})^2}$$

$$J_{21} = 0,$$

$$J_{22} = -x_0^1.$$

Note that the following always holds

$$\frac{2e^{\mu x_0^1 - g_0}}{(1+e^{\mu x_0^1 - g_0})^2} < \frac{1}{2},$$

which means that for $\mu < 2$,

$$\frac{2e^{\mu x_0^1 - g_0}}{(1+e^{\mu x_0^1 - g_0})^2} < \frac{1}{\mu},$$

hence $J_{11} < 0$. But given that $x_0^1 < 0$, $J_{22} > 0$ hence

$$|J| = x_0^1 \left[-1 + \frac{2\mu e^{\mu x_0^1 - g_0}}{(1+e^{\mu x_0^1 - g_0})^2} \right] > 0$$

proving that the stationary equilibrium is a saddle point.

Similarly for the $\{x_1^1, 1\}$ equilibrium, the elements of the Jacobian are as follows

$$J_{11} = -1 - \frac{e^{\mu x + 1 - g_0}(-1 + e^{\mu x + 1 - g_0})\mu}{(1+e^{\mu x + 1 - g_0})^2} + \frac{e^{\mu x + 1 - g_0}\mu}{1 + e^{\mu x + 1 - g_0}}$$

or,

$$J_{11} = -1 + \frac{2\mu e^{\mu x_1^1 + 1 - g_0}}{(1+e^{\mu x_1^1 + 1 - g_0})^2}$$

$$J_{12} = \frac{2e^{\mu x_0^1 + 1 + g_0}}{(e^{\mu x_0^1 + 1 + g_0} + e^{g_0})^2}$$

$$J_{21} = 0,$$

$$J_{22} = x_1^1.$$

Note that given that $J_{22} > 0$, as in the previous case, it is trivial to prove that $\{x_1^1, 1\}$ is also a saddle equilibrium.

$\square$

Proof of Corollary 1

The discriminant of the Jacobian (21) at the centrist equilibrium is

$$\Delta = [Tr(J)]^2 - 4|J| = \left(\frac{\mu - 2}{2}\right)^2 + 2(g_0^2 - g_0),$$

which means that $\Delta < 0$ if and only if

$$(\mu - 2)^2 < 8(g_0 - g_0^2),$$

or equivalently if

$$-8(g_0 - g_0^2) < \mu - 2 < 8(g_0 - g_0^2),$$

or,

$$2 - 2\sqrt{2(g_0 - g_0^2)} < \mu < 2 + 2\sqrt{2(g_0 - g_0^2)}.$$

$\square$

Proof of Proposition 2

The proof follows the first part of Proposition 1, but for $\mu > 2$. It is sufficient to show that there exists a local maximum of $F(x)$, which is increasing in μ and that for some values of μ, this is positive while for others this is negative. Substituting μ, $F(x)$ is

$$F(x) = e^{2(\mu x - g_0)} - \frac{1+x}{1-x},$$

and also

$$F'(x) = -1 + 2\mu \frac{e^{\mu x + g_0}}{(e^{\mu x} + e^{g_0})^2}.$$

$F'(x) = 0$ for

$$x_a = \frac{g_0 - ln[\mu - 1 - \sqrt{(\mu - 2)\mu}]}{\mu}$$

or

$$x_b = \frac{g_0 - ln[\mu - 1 + \sqrt{(\mu - 2)\mu}]}{\mu}$$

Note that

$$F''(x) = \frac{2e^{\mu x + g_0}(e^{g_0} - e^{\mu x})\mu^2}{(e^{g_0} + e^{\mu x})^3}$$

then for x_a to be a local max, the following should hold

$$F''(x_a) = -\frac{2\mu^2 \left[1 - \mu + \sqrt{(\mu - 2)\mu}\right] \left[2 - \mu + \sqrt{(\mu - 2)\mu}\right]}{\left[-\mu + \sqrt{(\mu - 2)\mu}\right]^3} < 0.$$

Note that $-\mu + \sqrt{(\mu - 2)\mu} < 1 - \mu + \sqrt{(\mu - 2)\mu} < 0$ and $2 - \mu + \sqrt{(\mu - 2)\mu} > 0$. Hence

$$F''(x_a) < 0.$$

Substituting x_a to F, gives,

$$F(x_a) = \frac{-g_0 + \sqrt{(\mu-2)\mu} + ln[\mu - 1 - \sqrt{(\mu-2)\mu}]}{\mu}. \tag{22}$$

Then

$$\frac{\partial F(x_a)}{\partial \mu} = \frac{g_0 - ln[\mu - 1 - \sqrt{(\mu-2)\mu}]}{\mu^2}.$$

Note that $\mu - 1 - \sqrt{(\mu-2)\mu} < 1$ as

$$(\mu-2)^2 < (\mu-2)\mu$$

and

$$\mu - 2 < \mu.$$

Hence $\frac{\partial F(x_a)}{\partial \mu} > 0$.

In order to complete the proof of parts (i) and (ii) it is sufficient to show that there exists $\mu = \bar{\mu}(g_0)$ for which

$$F(x_a) = 0$$

For the above to hold the following should hold

$$\sqrt{(\mu-2)\mu} + ln[\mu - 1 - \sqrt{(\mu-2)\mu}] = g_0 \tag{23}$$

Note that the derivative of the numerator of (22) is $\frac{\sqrt{(\mu-2)\mu}}{\mu} > 0$ and that for $\mu = 2$,

$$F(x_a) = \frac{-g_0}{2} < 0$$

while for $\mu = 3$, $F(x_a) > 0$. Hence there exists $\bar{\mu}(g_0)$ which is the solution of (23) such that (i) and (ii) are true.

Note that we can express $\Phi(x)$ as

$$\Phi(x) = e^{2\mu x + 1 - g_0]} - \frac{1+x}{1-x}$$

hence

$$\Phi'(x) = -\frac{e^{2g_0} + e^{2(1+\mu x)} - 2(\mu-1)e^{1+g_0+\mu x}}{(e^g + e^{1+\mu x})^2}$$

$\Phi'(x) = 0$, for

$$x_c = -\frac{1 - g_0 + ln[\mu - 1 - \sqrt{(\mu-2)\mu}]}{\mu}$$

or

$$x_d = -\frac{1 - g_0 + ln[\mu - 1 + \sqrt{(\mu-2)\mu}]}{\mu}.$$

Note that

$$\Phi''(x) = \frac{2e^{1+g_0+\mu x}(e^{g_0}e^{1+\mu x})\mu^2}{(e^{g_0}e^{1+\mu x})^3}$$

which gives

$$\Phi''(x_c) = -\frac{2\mu^2\left[1 - \mu + \sqrt{(\mu-2)\mu}\right]\left[2 - \mu + \sqrt{(\mu-2)\mu}\right]}{\left[-\mu + \sqrt{(\mu-2)\mu}\right]^3},$$

and

$$\Phi''(x_d) = \frac{2\mu^2\left[\mu - 1 + \sqrt{(\mu-2)\mu}\right]\left[\mu - 2 + \sqrt{(\mu-2)\mu}\right]}{\left[\mu + \sqrt{(\mu-2)\mu}\right]^3}.$$

Note that $-\mu + \sqrt{(\mu-2)\mu} < 1 - \mu + \sqrt{(\mu-2)\mu} < 0$ and $2 - \mu + \sqrt{(\mu-2)\mu} > 0$. Hence $\Phi''(x_c) < 0$ and $\Phi''(x_d) > 0$, which means that at x_c there is a local minimum of $\Phi(x)$. Substituting x_d, we get

$$\Phi(x_d) = \frac{1 - g_0 - \sqrt{(\mu-2)\mu} + ln[\mu - 1 - \sqrt{(\mu-2)\mu}]}{\mu}$$

with

$$\frac{\partial \Phi(x_d)}{\partial \mu} = \frac{g_0 - 1 - ln[\mu - 1 + \sqrt{(\mu-2)\mu}]}{\mu^2}.$$

Given that $\mu > 2$, we get that $\frac{\partial \Phi(x_d)}{\partial \mu} < 0$. Hence in order to complete the proof it is sufficient to show that there exists $\hat{\mu}(g_0)$ such that for $\mu = \hat{\mu}(g_0)$, $\Phi(x_d) = 0$.

Note that for $\mu = 2$,

$$\Phi(x_d) = \frac{1 - g_0}{2} > 0$$

while for $\mu = 3$, $\Phi(x_d) < 0$.

□

Proof of Proposition 3

As shown in the proof of Proposition 2, $\bar{\mu}(g_0)$ and $\hat{\mu}(g_0)$ are the solutions of $F(x_a) = 0$ and $\Phi(x_d) = 0$, respectively. Given that $\mu > 2$ these can be alternatively expressed as

$$g_0 = \sqrt{(\mu - 2)\mu} + ln[\mu - 1 - \sqrt{(\mu - 2)\mu}], \tag{24}$$

and

$$g_0 = 1 - \sqrt{(\mu - 2)\mu} + ln[\mu - 1 - \sqrt{(\mu - 2)\mu}]. \tag{25}$$

For 24

$$\frac{dg_0}{d\mu} = \frac{\sqrt{(\mu - 2)\mu}}{\mu} > 0.$$

Using the inverse function rule, this implies that for $\mu = \bar{\mu}(g_0)$, $\frac{\partial \bar{\mu}(g_0)}{\partial g_0} > 0$.For 25

$$\frac{dg_0}{d\mu} = -\frac{\sqrt{(\mu - 2)\mu}}{\mu} < 0,$$

which means that $\frac{\partial \hat{\mu}(g_0)}{\partial g_0} < 0$. Hence the first part of the proposition is proven.

Note that for $g_0 = \frac{1}{2}$, $F(x_a) = -\Phi(x_d)$, which proves the second part of the proposition.

□

Figures

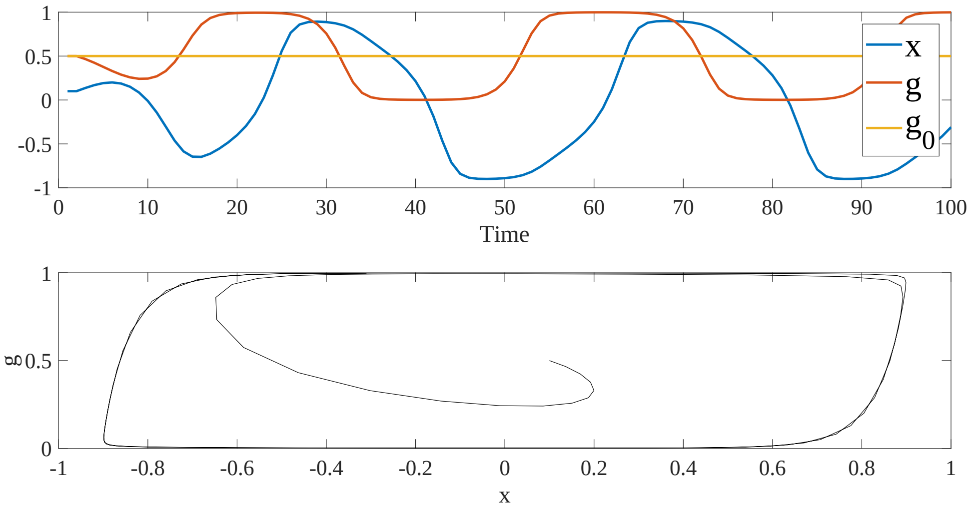

Figure 1: Single run of the model with exogenous income. Parameter set: $g_0 = 0.5; \mu = 2.4$.

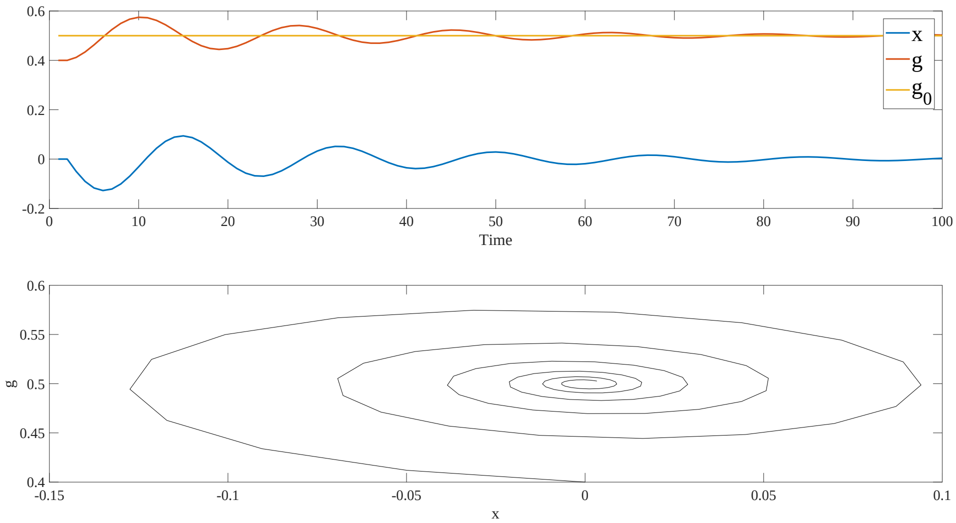

Figure 2: Single run of the model with exogenous income. Parameter set: $g_0 = 0.5; \mu = 1.8$

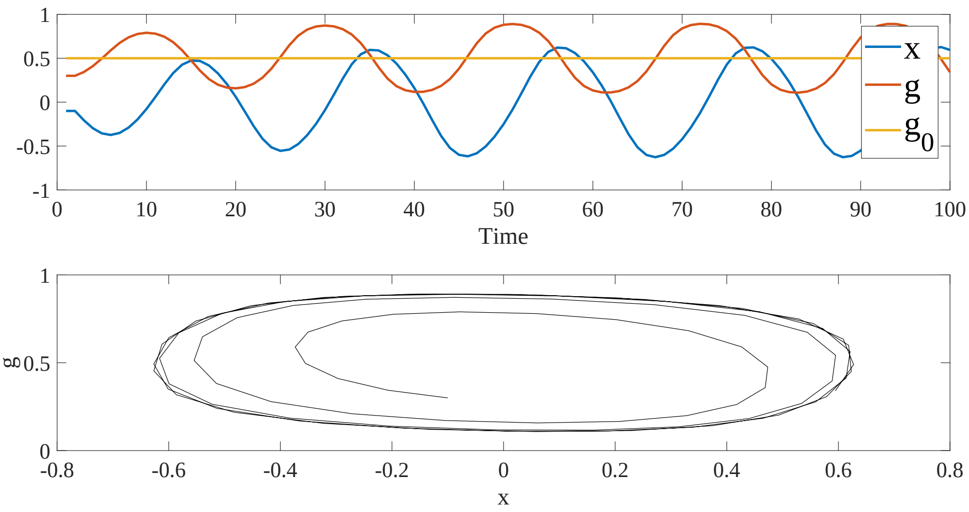

Figure 3: Single run of the model with exogenous income. Parameter set: $g_0 = 0.5; \mu = 2$; initial conditions: $x = -0.1; g = 0.3$.

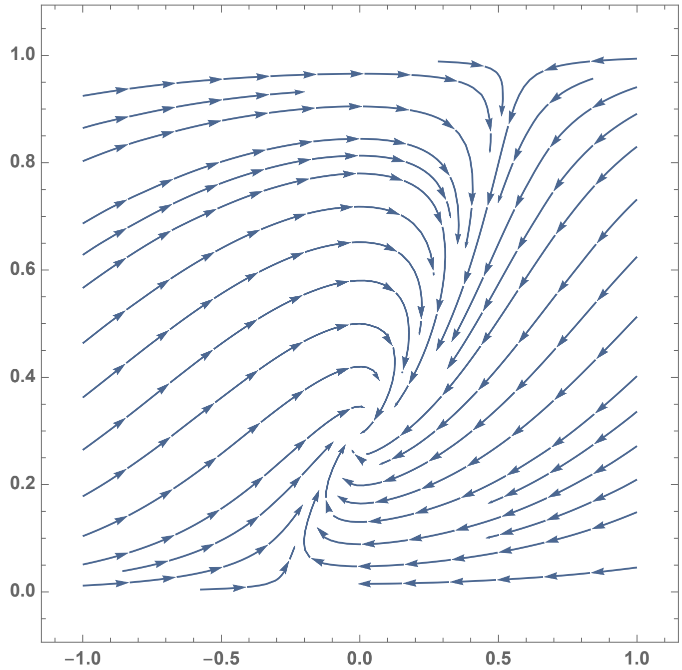

Figure 4: Phase diagram for x, g with $g_0 = 0.3$, $\mu = 1$

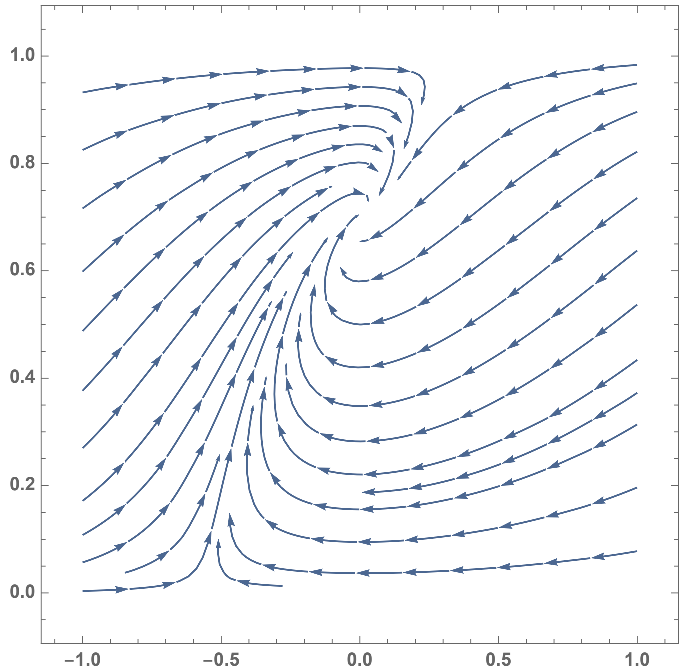

Figure 5: Phase diagram for x, g with $g_0 = 0.7$, $\mu = 1$

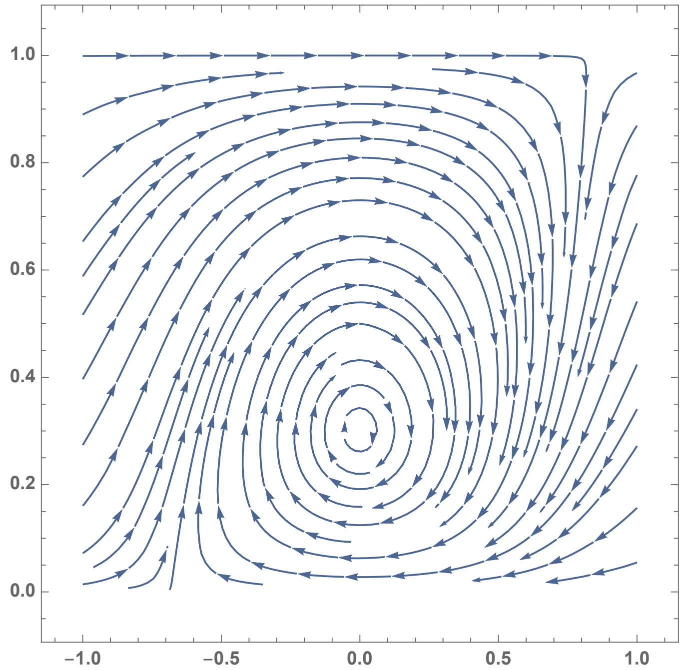

Figure 6: Phase diagram for x, g with $g_0 = 0.3$, $\mu = 2$

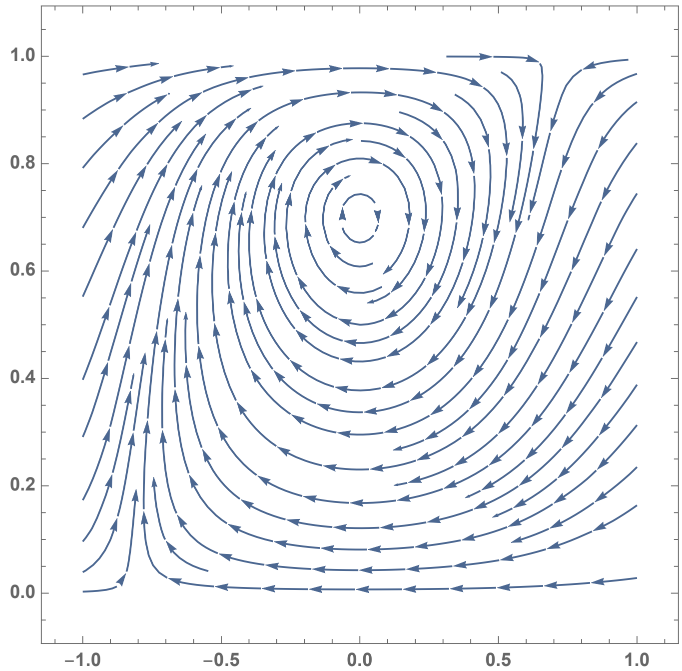

Figure 7: Phase diagram for x, g with $g_0 = 0.7$, $\mu = 2$

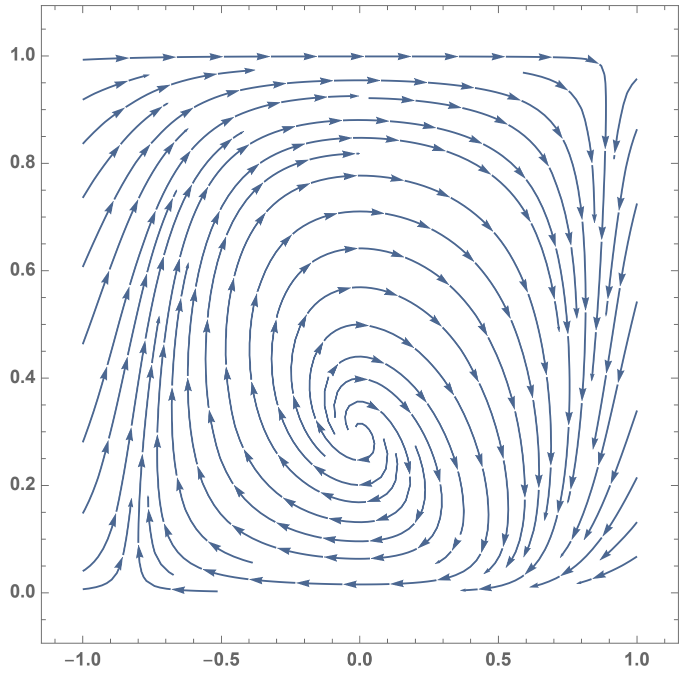

Figure 8: Phase diagram for x, g with $g_0 = 0.3$, $\mu = 2.5$

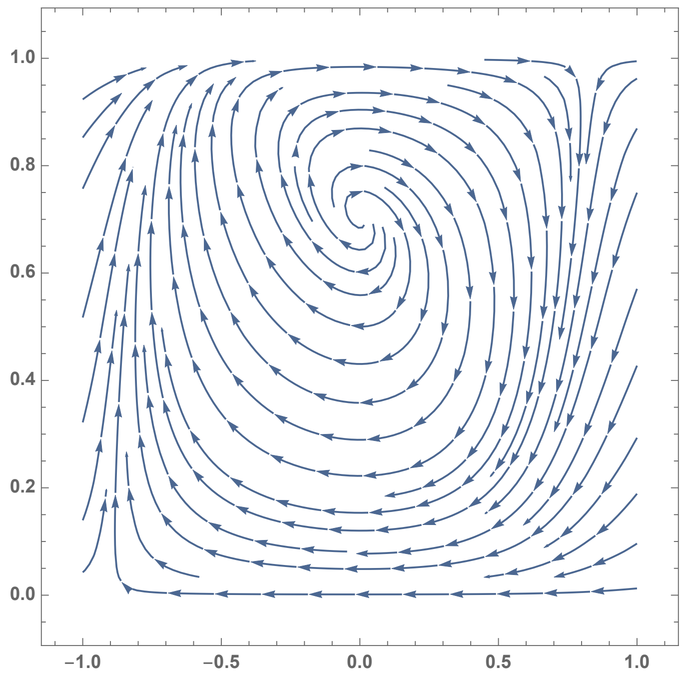

Figure 9: Phase diagram for x, g with $g_0 = 0.7$, $\mu = 2.5$

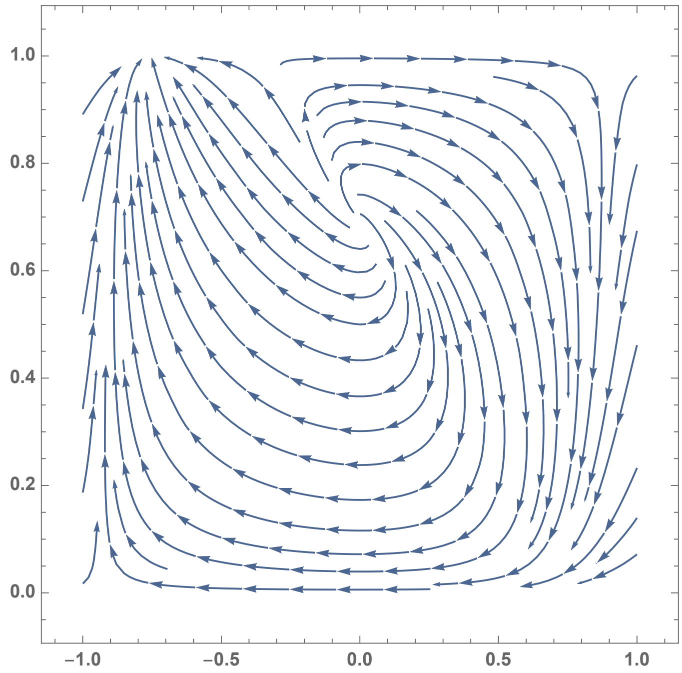

Figure 10: Phase diagram for x, g with $g_0 = 0.7$, $\mu = 3$

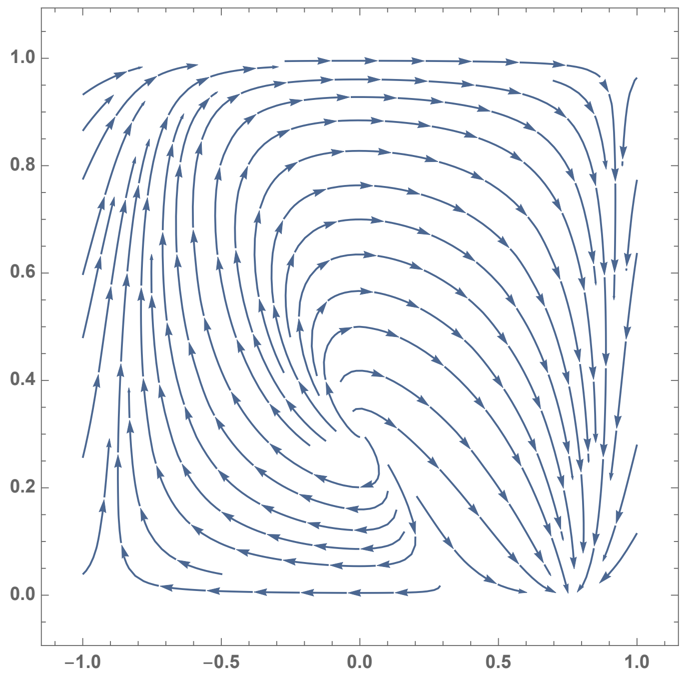

Figure 11: Phase diagram for x, g with $g_0 = 0.3$, $\mu = 3$

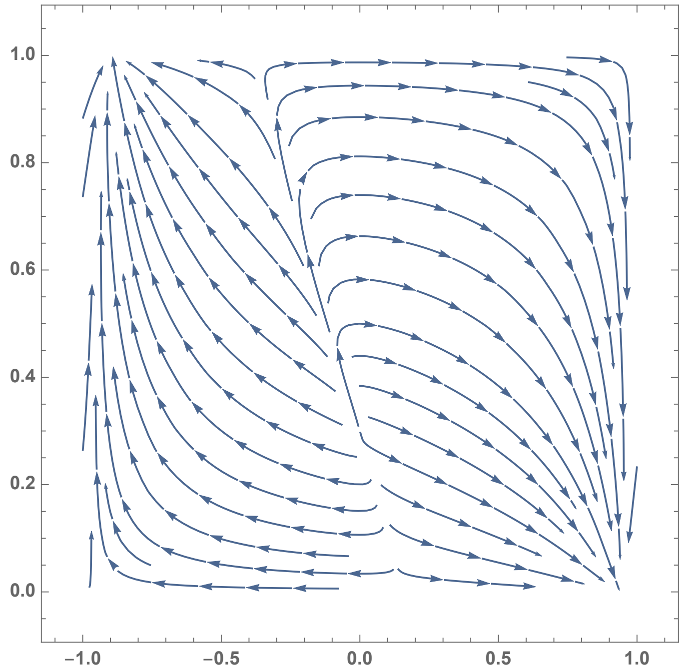

Figure 12: Phase diagram for x, g with $g_0 = 0.3$, $\mu = 4$

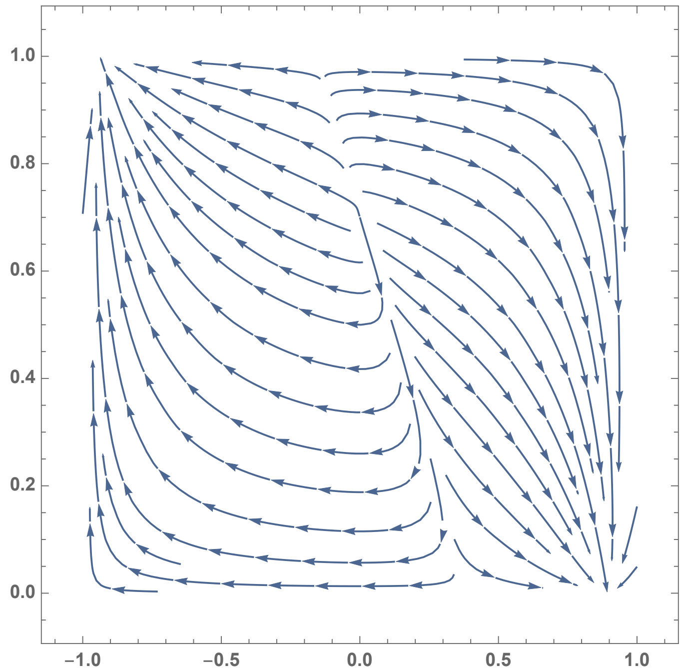

Figure 13: Phase diagram for x, g with $g_0 = 0.7$, $\mu = 4$

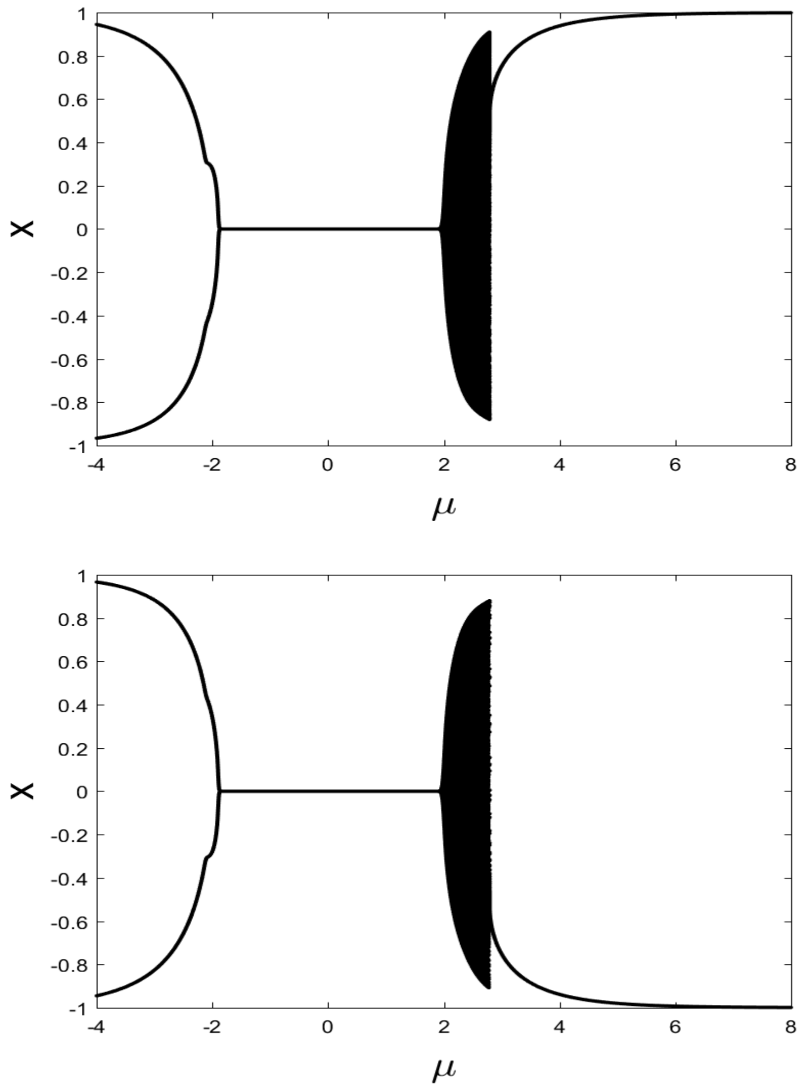

Figure 14: Bifurcation plot for μ and x with $g_0 = 0.3$ (upper panel) and $g_0 = 0.7$ (lower panel).

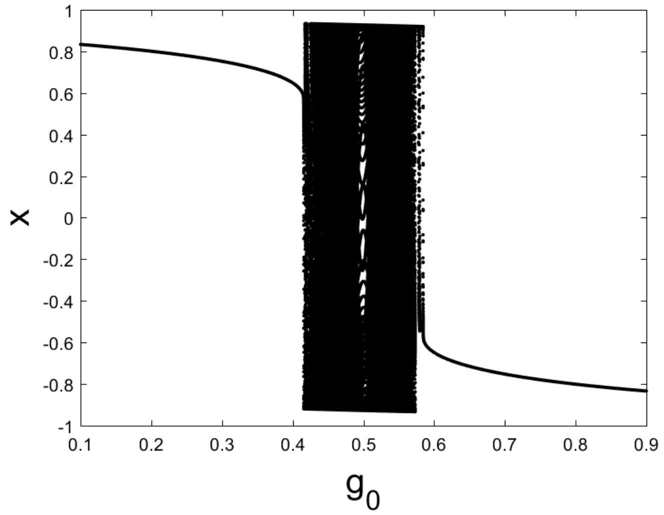

Figure 15: Bifurcation plot for g_0 and x with $\mu = 3$.

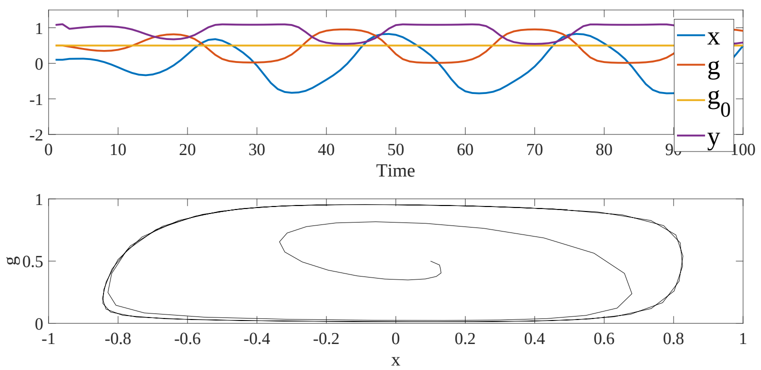

Figure 16: Single run of the model with endogenous income (all variables in levels). Parameter set: $\beta_x = 2.5; \beta_y = 1.5; g_0 = 0.5; \gamma_1 = -0.0001; \gamma_2 = 0.0024; \gamma_3 = .077$.

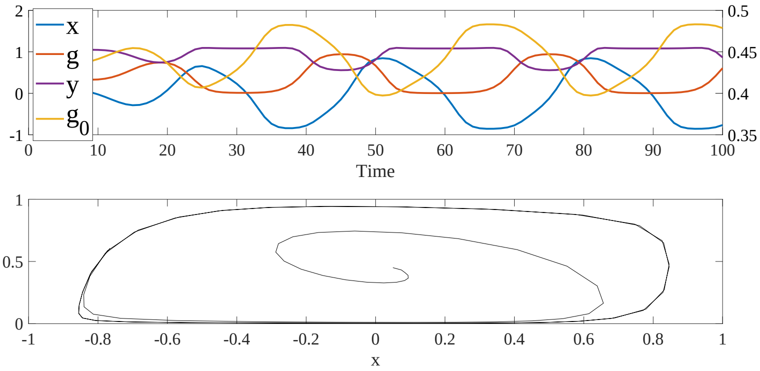

Figure 17: Single run of the model with endogenous income and endogenous g_0 (g_0 scale on the right axis, all variables in levels). Parameter set: $\beta_x = 2.5; \beta_y = 1.5; \gamma_1 = -0.0001; \gamma_2 = 0.0024; \gamma_3 = .077$.

Parameter	Subcase	Equilibria
$\mu \leq 2$		$\{0, g_0\}$, $\{x_0^1, 0\}$, with $x_0^1 \in (-1, 0)$ and $\{x_1^1, 1\}$, $x_1^1 \in (0, 1)$
$\mu > 2$	$\bar{\mu}(g_0) > 2$	$\{x_0^3, 0\}$, with $x_0^3 \in (0, 1)$, if $\mu = \bar{\mu}(g_0)$
		$\{x_0^2, 0\}$, with $x_0^2 \in (0, 1)$ and $\{x_0^3, 0\}$, with $x_0^3 \in (0, 1)$, if $\mu > \bar{\mu}(g_0)$
	$\hat{\mu}(g_0) > 2$	$\{x_1^3, 1\}$, with $x_0^3 \in (-1, 0)$, if $\mu = \hat{\mu}(g_0)$,
		$\{x_1^2, 1\}$ with $x_1^2 \in (-1, 0)$, and $\{x_1^3, 0\}$, with $x_1^3 \in (-1, 0)$, if $\mu > \hat{\mu}(g_0)$

Table 1: Summary of the different equilibria from Propositions 1,2.

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