Economic Structures and Dynamics: A Morphogenetic View*

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Abstract

Economic systems are characterised by constant change and evolution, and explanations concerning the properties of economic structures have received sustained interest. The structure of a system and its dynamics can influence each other through feedback effects. In this paper we offer a brief survey of how structure plays a role in dynamic economic theory – in particular, growth and business cycles. We propose a morphogenetic framework, inspired from the creation of forms in developmental biology, as a potential unifying approach for studying economic structures and their dynamics. We synthesise insights from three different strands of research, focusing on the role of coupling, diffusion and symmetry-breaking. We highlight their existing and prospective links with economics.

Keywords: Economic dynamics; Morphogenesis; Turing instabilities; Symmetry-breaking; Structural Change

1 Introduction

Economic systems are far from stable, uniform or homogenous. The same applies to the processes by which they evolve over time and space, presenting a wide range of patterns that merit investigation, including: inhomogeneous development across countries and regions, sustained oscillations in aggregate output and growth, financial instability

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that punctuates long periods of relative stability, spatio-temporal patterns of innovation and technological change, skewed distributions of wealth, income, evolving patterns of demand and consumption. These patterns have clear links with the structural properties of economies, but unearthing the exact mechanisms that guide their formation is both challenging as well as a topic of sustained interest.

The term 'structure' has wide-ranging interpretations across different disciplines, and any definition specific to the discipline of economics must recognise the characteristics unique to economic structures. These include the time irreversibility of changes and the complexity of their dynamics. Technological change, for instance, acts as a disruptive force with cumulative effect on economic structures, and is mostly unidirectional. As economic structures grow and evolve, they also display increasing complexity. They display neither complete stability nor instability, and they undergo significant transformations in composition and organisation at gradual or sudden rates of change. Theories of structural change in economics are geared towards identifying the underlying mechanisms and processes that explain these patterns, but the concept of structure is relevant for economic theory more widely as well. The latter aspects include: formulating methods for comparing different structures, developing a notion of *structural equivalence*, identifying the extent of self-regulating capabilities, understanding structural stability, complexity, drivers of transformation, and tools to study their potential and actual dynamics.

Historically, these questions have been studied separately across different branches of economics. What is arguably lacking is a unified approach to investigate diverse questions concerning economic structure and dynamics. The main thrust of this article is to explain why morphogenetic frameworks – an idea which originates from developmental biology – can serve as a potential candidate. Our suggestion is that this framing provides a broad umbrella for the unified study of structure, transformation paths, dynamics, complexity and evolution. To this effect we discuss three traditions to investigating morphogenetic frameworks and their links with economic thinking: (i) Turing's model of morphogenesis in developmental biology, (ii) Fermi-Pasta-Ulam-Tsingou approach to numerically investigating thermalisation in solids, (iii) Cellular automata tradition, also one of the origins of agent-based modelling. Structure and dynamics play a prominent role in each of these, and there are significant variations in how directly these ideas can, and have historically been applied to understand economic structures.

In this paper, we focus on theoretical approaches to model structure and dynamics. Empirical issues concerning these topics such as econometric methods for detecting structural change, though important, are outside the scope of this paper. The remainder of the paper is organised as follows: §2 introduces structure, dynamics, complexity and their interconnections from a mathematical point of view. §3 offers an overview of the term structure in economics, surveying how growth, development and business cycles have been conceptualised. §4 introduces approaches to morphogenetic frameworks in the three traditions, while §5 synthesises core insights from these models and, together with coupling and diffusion, their application in economics. §6 concludes.

2 Structures, transformations and complexity

> As a first approximation, we may say that a structure is a system of transformations. Inasmuch as it is a system and not a mere collection of elements and their properties, these transformations involve laws: the structure is preserved or enriched by the interplay of its transformation laws, which yield results external to it. In short, the notion of structure is comprised of three key ideas: the idea of wholeness, the idea of transformation and the idea of self-regulation [Piaget, 1971, p.5]

The term 'structure' is frequently understood as a loose description of relationships between parts and the whole. However, it is used to refer to a wide range of concepts across different disciplines such as physics, psychology, mathematics, linguistics, biology, ecology and social sciences. It is quite challenging to have a definition that covers all these different usages in a consistent manner. Piaget [1971] has attempted to synthesise some common threads and his tripartite characterisation of this notion mentioned above can serve as a useful heuristic for the purpose of this paper. Closely associated with structure is the idea of transformation, and in particular, dynamics. One can view dynamics as the transformation of a given structure over time. Furthermore, we can speak of the structure of solutions or trajectories of a system evolving over time. Several questions concerning structure may be of interest across different domains: the extent of invariance of a structure under various transformations, properties that can or cannot be deduced from a given structural specification, stability, symmetry, to mention a few. Similarly, dynamics presents a whole set of issues such as appropriate formalisms to encapsulate dynamic behaviour, characterisation of its long term attractors, their uniqueness properties, transitional paths, stability, equivalence and so on. Complexity is an overarching idea which applies to both structure and dynamics. For instance, descriptive complexity of structures, dynamic complexity, and computational complexity. Clearly, it would be useful to integrate these interrelated themes within a unified framework.

In terms of formal methods, there are distinct fields of mathematics that have been developed over the years. Structure has been a central theme in group theory, category theory, topology, measure theory, model theory (mathematical logic) among other fields. In the case of dynamics, ordinary and partial differential equations, difference equations, numerical analysis and dynamical systems theory have been developed to study qualitative and quantitative aspects. It is intuitive to expect that in study of structures, the idea of equivalence is central. One needs to map properties between various structures and should be able to conclude whether they are equivalent, alike or different. The exact notion of formal equivalence will obviously vary based on the field of study, the nature of structure and space. In general, the idea of *morphism* (a generalisation of homomorphism) plays a central role for understanding equivalence in category theory. If we consider a category of topological spaces, continuous functions play the role of morphisms. We now describe some of these ideas in the context of dynamical systems since it will be useful for our subsequent discussion on structural change, bifurcation and symmetry-breaking.

2.1 Structural equivalence

Consider two continuous time dynamical systems with smooth right-hand sides[1]

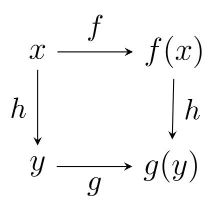
$$\dot{x} = f(x), \quad x \in \mathbb{R}^n \tag{1}$$

$$\dot{y} = g(y), \quad y \in \mathbb{R}^n \tag{2}$$

Let ϕ^t and ψ^t denote the corresponding flows.

Definition 1. *A dynamical system $\{T, \mathbb{R}^n, \phi^t\}$ is called topologically equivalent to another dynamical system $\{T, \mathbb{R}^n, \psi^t\}$, if there is a homeomorphism $h : \mathbb{R}^n \to \mathbb{R}^n$ mapping orbits of the first system onto orbits of the second system, preserving the direction of time.*

A homeomorphism is an invertible map such that both the map and its inverse are continuous. Topological equivalence implies that if x and y are related by the homeomorphism h, $y = h(x)$, then the first orbit is mapped onto the second one by this map h.



Let $y = h(x)$ be an invertible map $h : \mathbb{R}^n \to \mathbb{R}^n$, which is *smooth* together with its inverse (i.e., h is a diffeomorphism) and such that, for all $x \in \mathbb{R}^n$, we have

$$f(x) = M^{-1}(x) g(h(x)), \tag{3}$$

where, M denotes the Jacobian matrix of $h(x)$ evaluated at the point x.

Definition 2. *Two dynamical systems are called smoothly equivalent, if they satisfy (3) for some diffeomorphism h.*

Let us now consider a parametrised dynamical system

$$\dot{x} = f(x, \alpha), \quad x \in \mathbb{R}^n, \alpha \in \mathbb{R}^m \tag{4}$$

As the parameters vary, the dynamical system may or may not be topologically equivalent in terms of the phase portraits.[2] The appearance of non-equivalent phase portraits

[1]The discussion on structural equivalence and morphogenesis has been predominantly been in relation to differential equations. We will focus largely on differential equations and continuous time dynamical systems in this paper. For definitions related to smooth equivalence concerning discrete dynamical systems and for local topological equivalence, see Kuznetsov [2010, ch. 2].

[2]We note that whether a dynamical system is topologically equivalent to another may not always be *effectively* (i.e., algorithmically) decidable in general. We thank the anonymous referee for pointing this out. A detailed discussion of computability issues in relation to topological spaces and dynamical systems, while important, are outside the scope of this paper.

due to variation of parameters is called a bifurcation. It marks a qualitative change in the dynamics associated with parametric variations and can also be seen as a failure of the structural stability. Based on Kupka-Smale conditions, bifurcations can be classified as local (e.g., Andronov-Hopf, saddle-node, Neimark-Sacker) or global (e.g. homoclinic, heteroclinic) as well as according to the number of parameters that vary (i.e., codimension). For instance, as the parameter α is the above dynamical system varies, the stable node or a focus (equilibrium point) can become unstable and give birth to a limit cycle. In this case, these bifurcations can be supercritical or subcritical based on stability of the associated limit cycle. In general, bifurcation theory works with a *given* structure (parametrised dynamical system) and develops methods to investigates bifurcation that happen within a family of solution trajectories associated with that structure.

Structural change on the other hand, refers to non-trivial changes that have implications for some or other properties in structure. At times this is narrowly interpreted as changes in the proportions and ratios associated with different sub-parts. However, in some situations a mere change may not be sufficient, but it should also be non-trivial. Bifurcations described above might capture some aspects of structural change. Growth often poses interesting questions concerning feedback influences on the original structure. These feedback effects can be self-regulatory or destabilising to varying degrees. The extent to which these feedback effects render the structure invariant is an important aspect of studying structural change, and it is intertwined with the notion of symmetry. In the case of biological organisms, growth can lead to symmetry-breaking as the organism takes on definite shapes and forms. Finally, the irreversibility of time can also introduce an interesting dimension while studying these issues. In the following section we provide a selective overview of how the terms structure and dynamics are used in economic theory.

3 Structure and dynamics in economics: A bird's-eye view

We begin with a brief overview of how the term structure is understood and used in different branches and schools. Thereafter, we outline the ways in which change or transformation of economic structures has been conceptualised.

3.1 Varieties of structure in economics

The notion of structure has been an integral part of economic theorising right from its origins in pre-classical thinking. With the emergence of a systemic vision in the eighteenth century, starting from Richard Cantillon and through the physiocrats, an economy was seen as an integrated system composed of different sectors, markets or classes. Their theories dealt with the circular flow of income, expenditure and labour between these subcomponents of the system, the mechanisms that underpin balance in these flows, and policies that lead to a supposed *natural state*. The idea of perpetual reproduction of the structure and associated questions of value and relative prices were at the core of their analyses. A quintessential expression of this way of representing and analysing an economic system can be seen in François Quesnay's *Tableau Économique*([1759] 1972).

Among the classical economists, a structural approach was evident in Adam Smith's analysis of the factors that determine the wealth of nations. For Smith, the division of labour that generates productivity gains, which in turn alters the structure of employment (i.e., the allocated proportions of productive labour) is the key driver of economic expansion. For Ricardo, an alteration of the relative proportions of different components in the economic system through distribution was central to increasing production. The presence of structural thinking in Marx is evident too with his conceptualisation of social structures. In particular, his analysis of production, class structures in capitalism, the organic composition of capital and circuits of commodity production are all striking examples. An emphasis on the notion of *proportions* in relation to the aggregate structure, its consequent influence on the creation of surplus and thus on the expansion of economic system are all hallmarks of the classical approach. The physiocrats and most classical economists also believed in the self-regulating capabilities of the economic system.[3]

Although classical economists focused on structural characteristics and the interrelationships between respective components, their emphasis was largely on the proportionate or uniform expansion of economic systems. They paid relatively less emphasis to the intricacies concerning alterations in structure caused by growth, or in other words, to structural dynamics [Pasinetti and Scazzieri, 1987]. Similarly, the circular flow models of the classicals did not substantively focus on the time structure of economic processes either.

In the marginalist tradition, von Thünen deserves a special mention for his work on how economic forces shape spatial structures by influencing land use patterns over time. Among the neoclassicals, Walras in particular focused on the interconnections between different markets and strived to solve for the exchange ratios that would establish an overall systemic equilibrium. Preferences, endowments and technology (the neoclassical closure) jointly define the structural properties of an economic system within this theory. In Walras' work on general economic equilibrium, despite claims about the role of tâtonnement processes, dynamics play little role [Velupillai, 2015]. Instead, Walras was devising solution methods to find prices (exchange ratios) that are associated with equilibria, but he did not study dynamic features such as the expansion of a system and its subsequent influence on the structure.

In modern usage, the term structure is used in economic theory in a variety of contexts: preference structures, endowment structures, time structure of production, structure of production networks, industrial structures, sectoral structures, capital structures, market structures, spatial structure of development, to name a few. Not all of these references to structure appeal to the underlying concept in a consistent manner, for instance as a way of specifying a relationship between parts and the whole. The notion of structure is also invoked at various, and in particular higher, levels of aggregation. For instance, it is legitimate to speak of the structure of relationships between variables even in the

[3]Robert Malthus was among the notable dissenters to this view.

highly aggregate representation of an economy.[4]

We will not attempt to provide an all-encompassing definition of structure here since this is outside the scope of the current paper, but it is still worth pointing out the distinction between static and dynamic structures. Static structures deal with a description of interrelationships between different variables or components of the system, or as describing patterns without an explicit time element. For example, input-output tables that capture a snapshot of horizontal interdependencies between different sectors in the economy.[5] In contrast, dynamic structures can be seen as the collection of possible time paths or trajectories associated with a given system. In the framework of the classicals and neoclassicals, economic theory was predominantly tied to the idea of equilibrium. Incorporating dynamic changes – and in particular crises and fluctuations in the economy – posed an important challenge. We will now turn to exploring this distinction in a bit more detail.

3.2 Structure and change in economics

While static characterisations are capable of providing informative descriptions of an economic system, we will mainly focus on dynamic structures that are used to characterise how systems evolve over time. Even a cursory understanding, it is safe to state that economic systems are characterised by a ceaseless process of transformation. Investigating the possibilities of these transformation processes is an integral part of any structural inquiry: the structure we endow in models of economies determines the nature of dynamics, the possibilities of evolution, and the implicit policy choices available to influence this direction.[6] Attempts to incorporate dynamic elements into economic theory that was hitherto predominantly static originate with Knut Wicksell and Irving Fisher's contributions around the turn of the twentieth century. These developments culminated in the birth of modern macroeconomics, in particular business cycle theory, theory of economic policy and growth theory. These attempts saw a major surge in theoretical innovations, particularly those in mathematical applications, through the inter-war year (particularly in the 1930s) and the immediate decades that followed the second world war. Major contributors include Frisch, Kalecki, Tinbergen, Keynes, Hayek, Myrdal, Lindhal, Hawtrey,

[4]Thus it may be erroneous to claim that structure is completely absent at high levels of aggregation, even though higher levels of disaggregation can yield a more nuanced understanding of a system. Potential loss of information that accompanies aggregation and the conditions on structure (of relationships) under which aggregation can be performed have been extensively researched in economics. A relevant example would be the conditions under which individual demand structures are preserved through aggregation, as outlined by Gorman Aggregation Theorem. Note that this is usually valid only for (quasi) homothetic preferences. Multi-sectoral models of structural change typically possess non-homotheticity assumptions of one form or other, thus reclaiming the role of disaggregated structure.

[5]It is however possible to endow time structure to these interdependencies, as the dynamic input-output literature specifies.

[6]It is possible that there may be multiple ways in which the same underlying system or structure can be represented [Pasinetti, 1973]. Even if they are formally equivalent, the choice of representation can have non-trivial implications in terms of the dynamic possibilities and potential actions within these different representations [Cardinale, 2018].

Aftalion, Schumpeter, Hicks, Harrod, Samuelson, Hansen, Leontief, Solow and Goodwin, among others.[7] For the present purpose, it is useful to focus on the development of business cycle and growth theories to illustrate the use of dynamic structures.

3.2.1 Business cycle theories

Business or trade cycle theory originated from attempts to incorporate fluctuations within the existing corpus of economic theory that was otherwise predominantly static and tied to the notion of equilibria. From the outset there were at least two competing visions of how this can be achieved. For Hayek, trade cycles needed merely to be reconciled with static-equilibrium economic theory rather than jettisoning the latter. However for Kuznets, static-equilibrium theory itself needed be abandoned. These contrasting viewpoints reflected diverging beliefs concerning the self-regulating capabilities of economic systems:

> *What [should] be discarded is the notion of a stable or slowly varying equilibrium and the equational system of solving economic problems.* What is substituted for it is a general recognition of the importance of the time element – a recognition which permits the utilization of the generalized experience of various special investigations in a more complex and a more realistic general theory of economic change. The equilibrium theory, in the limited meaning in which it is retained, will also be enriched, since the general theory of economic change will point out many more important economic factors than have heretofore been included in the equational systems of the mathematical school. *If we are to develop any effective general theory of economic change and any complete theory of economic behaviour, the practice of treating change as a deviation from an imaginary picture of a rigid equilibrium system must be abandoned* [Kuznets, 1930, p. 415; emphasis added] .

These developments themselves occurred against a backdrop of flourishing development in the mathematical theory of oscillations. In terms of the mathematics utilised, business cycle theories were formalised predominantly in terms of (linear or non-linear) ordinary differential, difference, or mixed difference-differential equations and later in terms of dynamical systems theory. Economic relationships formalised in such a manner were formulated as systems of equations, and the practise was to solve for or prove the existence of equilibrium solutions (or attractors).[8]

The choice of mathematical formalism was not trivial: the choice of models in the specification of structure decisively determines the dynamic possibilities associated with the economic system. Ragnar Frisch's early work on economic dynamics offers an illustrative example. In his foundational work on macrodynamics, Frisch [1933] developed a macrodynamic model in terms of mixed difference-differential equations to demonstrate

[7] See Ragupathy and Velupillai [2012] and references therein for more details.

[8] Note that the *limit cycles* that often characterise business cycles are tied to the notion of equilibria. They are merely a higher-dimensional analogue of fixed points.

the possibility of oscillations. He also makes a distinction between impulse and propagation mechanisms, which influenced subsequent approaches in equilibrium real business cycle theories. When subjected to exogenous impulses, the economic system is displaced from its equilibrium, but is supposed to return to equilibrium in an oscillating manner owing to its structure.[9] The role of structure may need a bit more elaboration: first, the choice of linear second-order differential equation in principle allows for oscillations. However, these oscillations can only be kept alive through a regular exposure to impulses in the form of erratic shocks external to the system. Second, the nature of characteristic roots associated with linear second-order differential equations (necessarily damped) implies that the effect of these shocks will dissipate and the system eventually returns to equilibrium. Thus the stability or the self-regulating capabilities of the economic structure were implicitly determined by the chosen mathematical formulation. As Samuelson wrote:

> In leaving Frisch's work of the 1930s on stochastic difference, differential and other functional equations, let me point out that a great man's work can, in its impact on lesser men, have bad as well as good effects. Thus, by 1940, Metzler and I as graduate students at Harvard fell into the dogma ... that all economic business-cycle models should have damped roots [...][W]hat was so bad about the dogma? Well, it slowed down our recognition of the importance of non-linear autorelaxation models of the van der Pol-Rayleigh type, with their characteristic amplitude features lacked by linear systems [Samuelson, 1974, p.10]

For instance, an alternative specification of an economy in terms of nonlinear differential equations allows for the possibility of sustained *endogenous* oscillations, i.e., endogenous instability of the economic structure.[10] Despite its beginnings as an endogenous theory, business cycle theory eventually came to be dominated by the exogenous approach in the form of Real Business Cycles. In this class of (neoclassical growth) models, business cycles are viewed as fluctuations in output (or employment) along a steady-state growth path. The baseline version of the model considers a representative agent endowed with rational expectations who maximises her utility, and the representative firm which maximises profit. The equilibrium is characterized by the paths of evolution of different variables (e.g. output, consumption and prices) in the steady state. In the presence of stochastic shocks, the equilibrium paths of evolution (known as the Dynamic Stochastic General Equilibrium or DSGE) for quantities and prices are characterized as stochastic processes. Here, studying business cycles translates into understanding how positive or negative stochastic shocks to a variable such as technology or productivity translates into output fluctuations. It may be reasonable to state that the scope of drastic changes to the original structure is rather limited.[11]

[9] See Zambelli [2007] for an in-depth discussion on the Frisch model and its shortcomings.

[10] For a detailed story of the origins and early development of endogenous business cycle theories in their mathematical mode, see Ragupathy and Velupillai [2012].

[11] The term 'scope' in this context is to be understood in terms of the notion of *relative invariance*:

3.2.2 Theories of growth, development and structural change

Growth or expansion of the economic system has been a topic of interest in several schools of economic thought, ranging from the classical, Marxian (reproduction schemes), neoclassical, evolutionary, Keynesian and structuralist approaches. Without attempting to provide an overview of the development of growth theory, in this subsection we will focus specifically on aspects that relate to structure and change.[12]

The process of economic expansion was explained in terms of mathematical models that encapsulated these dynamics. Early developments like the Harrod-Domar model focused on explaining the growth of aggregate income in an economy, and its relation to savings and productivity. von Neumann's [1945] growth model, which is difficult to classify within any single tradition, characterised the equilibrium state of an economy as that where production of all commodities grow at an equiproportionate rate. Influential contributions in this early period to growth theory by Solow and Swan were highly-aggregated single-sector models that relied on exogenous factors (such as technological progress, productivity) to explain growth. Disparities in income and growth across countries was attributed to differences in technology and the availability of capital per worker.

Later neoclassical models by Cass and Koopmans relaxed the assumption of constant savings rates in these models, introduced explicit micro foundations with optimising representative households and firms. The savings decisions were explicitly couched in terms of the preferences of these agents, thereby cementing an explicit link with competitive general equilibrium theory. This link gave rise to a focus on the optimality of these equilibria, and also provided scope for invoking the fundamental welfare theorems. The Ramsey-Cass-Koopmans model and Overlapping Generations Model both became the work horses of neoclassical growth theory. The problem of optimal growth was posed as one of deciding on the allocation of resources by the planner so as to maximise the utility of the household. The exogenous factors that drove growth in early models of growth were later endogenised with the introduction of investments in human capital, R&D, and associated spillover effects (in particular by the work of Romer and Lucas).[13] Multi-sector growth models with more disaggregated structures were again mostly concerned with optimal equilibrium growth paths, often within the context of planning.

Neoclassical growth theory (one, two, multi-sector, exogenous, endogenous, closed

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> "..the analysis of structural dynamics is associated with a general postulate of *relative invariance*, according to which any given economic system subject to an impulse or force is allowed to change its original state by following an adjustment path that belongs to a limited set of feasible transformations." [Landesmann and Scazzieri, 1990, p. 96]

The authors go on to explain why changes to equilibrium paths of evolution which arise from stochastic shocks to key variables are limited in scope, wherein a combination of a priori choices of allowing only certain structural-specification parameters to be variable in effect implies that only a limited set of transformations are feasible.

[12]We restrict our overview mostly to developments in growth theory in the post-war period, given our interest in the corresponding mathematical approaches.

[13]For an informative survey on the relationship between growth theory and theories of structural change in the historical context, see Gabardo et al. [2017].

and open economy versions) predominantly focuses on equilibrium configurations and steady states. Most models are geared towards solving for equilibrium growth paths, focusing on existence, uniqueness, stability, transitional dynamics and optimality of these paths. Stability of aggregate growth is taken as granted, and the models seek to explain this pattern of sustained growth. Part of the analysis focuses on testing for convergence or divergence in growth across different countries, and identifying the causes of these patterns. Structure, if at all it gets any attention, usually plays secondary role. The mathematical tools utilised in this framework are predominantly (linear and nonlinear) differential or difference equations, finite or infinite horizon optimisation (for instance by households and firms) solved using (deterministic or stochastic) dynamic programming or tools from optimal control.

In contrast to orthodox neoclassical growth theory's emphasis on balanced growth paths and the related transitional dynamics, the literature on development and structural change aims to understand the processes that accompany growth, and how the economic structure is itself transformed. The latter is often understood in terms of the composition of employment, output, sectoral structure of production and more broadly the organisation of the economy as a whole. The legitimacy of this focus can be justified as follows. First, there is no reason, *prima facie*, to view growth processes as being equilibrium phenomena. The stability of growth patterns is by no means universal across different countries, especially outside the developed world. Second, the dynamic forces that result in economic growth do not necessarily affect different parts of the economy in a proportionate manner. This unbalanced or non-uniform nature of growth at different levels and the accompanying drivers of these patterns are too crucial to be ignored. Third, there is a two-way effect at play as economies grow: the possibilities of growth are determined or constrained by the initial economic structure, and growth in turn affects the structure through feedback effects. Consequently, various kinds of interdependencies, dynamic adjustments and the role of structural aspects underlying growth all become relevant.[14] For example, the industrial revolution that unleashed a significant impetus for growth, resulted in important structural changes that in turn shaped the growth trajectory in subsequent years. Growth and structural transformation are thus highly interrelated.

[14]Such a broad, holisitic vision of growth–though still narrower then economic development which also includes social-political elements–is eloquently expressed in Kuznets [1966, p.1, italics added]:

> We identify the economic growth of nations as a sustained increase in per capita or per worker product, *most often accompanied by an increase in population and usually by sweeping structural changes.* In modern times these were changes in the industrial structure within which product was turned out and resources employed—away from agriculture toward nonagricultural activities, the process of industrialization; in the distribution of population between the countryside and the cities, the process of urbanization; in the relative economic position of groups within the nation distinguished by employment status, attachment to various industries, level of per capita income, and the like; in the distribution of product by use—among household consumption, capital formation, and the government consumption, and within each of these major categories by further subdivisions; in the allocation of product by its origin within the nation's boundaries and elsewhere; and so on.

Early investigations into structural aspects were largely empirical, including Clark [1940]; Fisher [1939]; Kuznets [1957]; Leontief [1936] and Chenery [1960]. Clark's early work pointed towards important factors that underpin structural change. These relate to *differential productivity growth* and *Engel effects*, which subsequent shaped a lot of theoretical research in this area. Leontief [1936, 1953] made important fundamental contributions in developing input-output methods to study economies and the vast amount of literature that it generated also played an important part in studying sectoral aspects of structural change. The focus of this literature was on explaining the falling employment share of labour in agriculture over time and the corresponding increase in manufacturing and services. The early literature in development economics took a broader view than growth theory, focusing relatively more on aspects of structural change. Kuznets, Hirschman, Nurske, Chenery, Lewis and Rosenstein-Rodan are amongst those who made important early contributions. Dual economy models in development concentrated on a range of asymmetries between the two sectors in an economy [Syrquin, 1988].

Explanations concerning structural change can be broadly divided into the following groups or combinations thereof: (a) the demand side view that focuses on the income elasticity of demand across different sectors; (b) the supply side view that focuses on technological factors and largely on productivity differences across different sectors. The demand side view crucially departed from mainstream growth theory in its discarding of the homothetic preferences assumption, whereby increases in income were accompanied with differential income elasticities.[15] On the supply side, non-homogenous nature of technological progress was the crucial factor which led to differential productivity growth across sectors [Gabardo et al., 2017].[16] There have also been attempts to integrate these two explanations within a single framework [Guilló et al., 2011].

It is easy to see that introducing a representative household into a model limits the scope for variation or heterogeneity in the structure of the underlying economy. And, with a representative household, characterising an economy through aggregation is then possible only under the restrictive conditions of the *Gorman Aggregation Theorem*. Gorman preferences imply linear Engel curves (depicting how the proportion of income spent on various goods change with variations in income) for each household (see Gorman [1961], Acemoglu [2009, §5.2]). In other words, the demand side of the economy remains unaffected by changes in the distribution of income. Many departures from aggregate, balanced growth theory in the neoclassical tradition routinely invoke non-homothetic pref-

[15]For a comprehensive treatment of structural change and Engel's law in the neoclassical framework, see Matsuyama [2019].

[16]See Syrquin [2012, p.72]:

> Once we abandon the fictional world of homothetic preferences, neutral productivity growth with no systematic sectoral effects, perfect mobility, and markets that adjust instantaneously, structural change emerges as a central feature of the process of development and an essential element in accounting for the rate and pattern of growth. It can retard growth if its pace is too slow or its direction inefficient, but it can contribute to growth if it improves the allocation of resources by, for example, reducing the disparity in factor returns across sectors or facilitating the exploitation of economies of scale

erences that violate the Gorman conditions.

Studies on structural change can also be broadly divided into neoclassical and non-neoclassical approaches. Within the neoclassical, a further categorisation follows from whether or not the models aim to situate structural change alongside balanced growth properties (so called Kaldor facts). Studies like Acemoglu and Guerrieri [2008]; Foellmi and Zweimüller [2008]; Ngai and Pissarides [2007] focus on explanations that reconcile structural changes with aggregate balanced growth. In contrast, studies like Baumol [1967]; Echevarria [1997]; Kongsamut et al. [2001]; Laitner [2000]; Matsuyama [1992] construct a range of models that explain structural change without relying on balanced growth. There is also now a considerable literature on the empirical divers of structural change [Van Neuss, 2019].

From a non-neoclassical perspective, there have been several contributions relating to structural change and growth. An important approach that focuses on changes in structure (composition) associated with lasting changes in economic magnitudes was put forward by Pasinetti [1983, 1993]. Changes in population, technological progress and demand patterns due to increasing income were at the core of his analysis. More specifically, changes in labour productivity brought about by varying levels of technological progress across sectors in turn affect patterns of demand, leading to adjustments in composition via prices and factor rewards. Consequent effect on attempts to maintain full employment, the coexistence of expanding and declining industries, and the dissonance between patterns of demand and output expansion were analysed within the framework of a vertically integrated production system.

The time structure of adjustments, and the viability and *traverse* of how the economic structure evolves have also been analysed within a neo-Austrian framework [Amendola and Gaffard, 1998; Hicks, 1973]. A useful classification of different analytical frameworks for analysing structural dynamics – horizontal interdependencies with and without time structure, varieties of vertical integration – can be found in Landesmann and Scazzieri [1990]. Schumpeterian approaches focus on the mechanics of innovation, their diffusion, emergence of new products, industries and uneven growth, which have been analysed in both neoclassical [Aghion and Howitt, 1992] and evolutionary frameworks [Filippetti et al., 2020; Saviotti and Pyka, 2004].

In what follows, we discuss a framework inspired from the creation of biological forms to think about the relationship between structure and dynamics. Our aim is to relate this to the ideas discussed so far in order to outline a unified perspective.

4 Structure and dynamics: A morphogenetic view

Morphogenesis is the branch of developmental biology concerned with how shapes are created as organisms grow. For example, how they develop from a simple egg into complex, full-fledged animals or plants. The subject has a long history dating back to at least the ancient Greek philosophers. The etymology of the term morphogenesis can be traced to its Greek roots; *morph* denotes 'form' and *genesis* means 'creation'. Thus, 'morphogenesis' refers broadly to the creation of forms, shapes or structures which accompanies a dynamic process of development. In biological systems, morphogenesis is often viewed as resulting from interactions between molecules and cells. However, even within a biological context, morphogenetic mechanisms depend not only on interactions, but also on various constraints and 'elaborate systems of feedback that regulate where and when those interactions take place' [Davies, 2013, p.8]. A central issue in biological systems is that growth of the different parts of the organism may not be uniform because sub-parts grow at different rates.[17]

It is fairly straightforward to see the potential relevance of morphogenesis to the study of economic structures and how they evolve. However, with the exception of infrequent allusions, to the best of our knowledge a detailed exploration of the connection and analogies between growth theory, structural change in economics and morphogenetic literature has yet to be undertaken.

Consider for instance morphometry, viz. the quantitative study of relationships between the size and shape of organisms and their variability.[18] As organisms grow and acquire definitive shape, questions concerning scaling become important. Two oft-invoked ideas in this context are *isometry* and *allometry*. Isometry refers to cases where the proportions between respective parts are preserved as growth takes place, whereas with allometric scaling this proportionality can change.

To appreciate the almost direct parallel with economic structures, consider Perroux's influential definition of economic structure as 'proportions and relationships that characterize . . . an economic setting in space and in time' [Pasinetti and Scazzieri, 1987]. Further, the view of economic growth as an uneven process across space and time is in fact characterised as the case in which structural proportions are not maintained [Ray, 2010]. There are other similarly striking parallels as well between isometric scaling and balanced growth paths.[19]

Unlike morphometry which focuses on the quantitative analysis of changes in proportions with growth, morphogenesis has broader scope, and seeks not only to characterise but also identify the *mechanisms* underpining the generation of shapes and form. Some notable works on morphogenesis include D'Arcy Thompson's "On Growth and Form" and Turing's paper on "The chemical basis of morphogenesis", which each adopt a different approach. Thompson relied heavily on geometric reasoning, whereas Turing largely resorted to algebra and calculus. At the risk of over-simplification, Thompson's approach can viewed as one in which structure or variations in shapes result from varying rates of growth in different directions. In contrast Turing can be seen as searching for a dynamic mechanism that underpins pattern formation, and below we now provide a brief review

[17] As Wolpert [2011, p.95] notes: "After 9 weeks of embryonic development, the head of a human embryo is more than a third of the length of the whole embryo, whereas at birth it is only about a quarter. After birth, the rest of the body grows much more than the head, which is only about an eighth of the body length in the adult."

[18] Some early pioneers of a geometric approach to morphometry based on proportions included artists such as Leonardo da Vinci and Albrecht Dürer, especially the latter's book *Vier Bücher von Menschlicher Proportion* (Dürer, [1528] 1969).

[19] For the history of morphometrics, see Reyment [2010].

of this approach.[20]

4.1 Turing Bifurcations

In an attempt to provide theoretical insights into morphogenesis, Alan Turing focused on the chemical basis of morphogenesis and showed that patterns can emerge from a previously homogenous, structureless state. These patterns emerge when two *morphogens* – form-creating substances – react amongst themselves and diffuse through the tissue. In such a reaction-diffusion system, Turing was able to mathematically demonstrate that even small, random disturbances can trigger off a process that eventually leads to loss of stability of the initial, symmetrical, structureless state. Thereby, this study proposed diffusion-induced symmetry-breaking as a plausible mechanism for pattern formation.

A notable feature of Turing's analysis is that he managed to demonstrate the counter-intuitive idea that two stabilizing influences could interact in a way that leads to instability. That is, a homogenous configuration of cells which is stable, loses its stability in the presence of diffusion (which is generally perceived to be a stabilizing factor) between the cells, thereby forming distinct patterns. Below, we present a schematic version of Turing bifurcation in a reaction-diffusion system.[21]

Consider a continuous ring of tissue, and let x and y be the morphogens and D_x, D_y be the respective diffusion coefficients.[22] We consider x and y as activator and inhibitor, respectively. An activator is a chemical that facilitates the growth in concentration of both chemicals, where as an inhibitor in contrast leads to a reduction in their concentrations. The rate of change of these morphogens is specified as below as a system of partial differential equations:

$$\begin{aligned} \frac{\partial x}{\partial t} &= f(x,y) + D_x \nabla^2 x \\ \frac{\partial y}{\partial t} &= g(x,y) + D_y \nabla^2 y \end{aligned} \tag{5}$$

Let us assume $x = x_0, y = y_0$ to be the (constant) values of x and y at the spatially homogenous state and $0 < D_x < D_y$. This condition means that the inhibitor diffuses faster than the activator ('short range activation and long range inhibition'), which is considered to be one of the general patterning principles in this system.

Consider a random disturbance that displaces the system from this homogenous state and by considering the system just outside the spatially homogenous state, we define $x = x_0 + \breve{x}$ and $y = y_0 + \breve{y}$ and $\breve{x}, \breve{y} \ll 1$. By linearising of the system in terms of $\breve{x}, \breve{y}$, we have,

[20]Strictly speaking, there is no growth in Turing's original published model and his main analysis concerned the formation of patterns and not shapes.

[21]The exposition closely follows Hoyle(2006), pp.12-21

[22]Instead, if we consider a ring of discrete cells then this will yield a system of ordinary differential equations. See section 6 of Turing [1952].

$$\begin{aligned}\frac{\partial \breve{x}}{\partial t} &= a\breve{x} - b\breve{y} + D_{\breve{x}}\nabla^2\breve{x} \\ \frac{\partial \breve{y}}{\partial t} &= c\breve{x} - d\breve{y} + D_{\breve{y}}\nabla^2\breve{y}\end{aligned} \tag{6}$$

The marginal reaction rates *a, b, c* and *d* are given as below:

$$a = \left.\frac{\partial f}{\partial x}\right|_{(x_0,y_0)}, b = -\left.\frac{\partial f}{\partial y}\right|_{(x_0,y_0)}, c = \left.\frac{\partial g}{\partial x}\right|_{(x_0,y_0)}, d = -\left.\frac{\partial g}{\partial x}\right|_{(x_0,y_0)}$$

By setting $\breve{x}$ and $\breve{y}$ as below, with x', y' as constants:

$$\breve{x} = x'e^{\lambda t}$$

$$\breve{y} = y'e^{\lambda t}$$

The associated marginal reaction rate matrix (Jacobian) of the system 6 is:

$$J = \begin{bmatrix} a & -b \\ c & -d \end{bmatrix}$$

The characteristic equation of this system is given by

$$\lambda^2 + (d-a)\lambda + (bc - ad) = 0$$

Its corresponding eigenvalues are the following:

$$\lambda_{1,2} = \frac{(a-d) \pm \sqrt{(a+d)^2 - 4bc}}{2}$$

If the conditions $a < d$ and $ad < bc$ are satisfied, then $x = x_0, y = y_0$ is the stable solution since both the eigenvalues are negative in that case. However, we are interested in analysing the nature of the solutions $\breve{x}, \breve{y}$ which are spatially varying and in the potential loss of stability that results from diffusion.

The solution $(\breve{x}(u,t), \breve{y}(u,t))$ of the system, which varies across space can be expressed in terms of Fourier series and we can analyse the Fourier modes.

$$\breve{x}(u,t) = x'e^{iku+\lambda t} + ..$$

$$\breve{y}(u,t) = y'e^{iku+\lambda t} + ..$$

where k is the wave vector. We can impose boundary conditions and obtain dispersion relation in terms of λ to analyse the stability of the linearised system. However, for simplicity, let us consider that the system is unbounded and ignore the higher order terms for $\breve{x}, \breve{y}$. Substituting the above in to the equation system 6, we can get an equation that relates the eigenvalues λ and the dispersion coefficients D_x, D_y and $k = |k|$, the norm of the wave vector:

$$\lambda^2 + \lambda(D_x k^2 + D_y k^2 - a + d) + (D_x k^2 - a)(D_y k^2 + d) + bc = 0$$

This is characteristic equation in terms of λ. We can examine the trace and determinant of the associated Jacobian to analyse conditions under which linear stability is violated. The trace $(D_x k^2 + D_y k^2 - a + d)$ cannot be negative given our assumptions about diffusion coefficients $D_x, D_y > 0$ and the assumption that $a < d$ (for the stability of the original spatially homogenous state). Therefore, the only possibility through which instability can arise is when the product of eigenvalues is negative.

$$\Lambda(k^2) \equiv (D_x k^2 - a)(D_y k^2 + d) + bc < 0$$

Using this we can find the value of k^2 for which minimum value of the product of the roots, Λ_{min}, is attained, which is:

$$k^2 = \frac{1}{2}\left(\frac{a}{D_x} - \frac{d}{D_y}\right)$$

Turing instabilities set in when $\Lambda_{min} < 0$ and $k^2 > 0$, that is:

$$\frac{a}{D_x} > \frac{d}{D_y}$$

Definition 3. ***Turing Instability***

A Turing instability, Turing bifurcation, or diffusion-driven instability occurs when a steady state, which is stable in the absence of diffusion, becomes unstable with the presence diffusion.

Remark 4. *Note that the factors that determine the loss of stability are totally intrinsic to the system, such as the diffusion rates and the reaction coefficients.*

At this juncture, it would be useful to contrast Turing bifurcation with another form of bifurcation used to model economic dynamics, viz. the Andronov-Hopf bifurcation theorem [Benhabib and Nishimura, 1979; Velupillai, 2006].

Definition 5. ***Andronov-Hopf Bifurcation***

Consider a one parameter, smooth dynamical system,

$$\dot{x} = f(x, \alpha), x \in \mathbb{R}^n, \alpha \in \mathbb{R}^1$$

As the parameter α is varied, if the hyperbolicity of equilibrium is violated due to a bifurcation that corresponds to the presence of $\lambda_{1,2} = \pm i\omega_0, \omega_0 > 0$, it is called Andronov-Hopf bifurcation.

Note that in this case, the bifurcation indicates the birth of a limit cycle from an equilibrium as the parameter is varied. For the case of continuous time dynamical systems, an equilibrium loses its stability when the eigenvalues cross the imaginary axis, due to which closed orbits bifurcate from the fixed point. Although both processes involve the loss of stability of an equilibrium, there are differences. Turing instabilities are associated with spatial patterns, while Hopf bifurcation relates to temporal oscillations. In the latter case, the topological equivalence or symmetry of the solution trajectories is broken. In the next two subsections, we outline two other approaches that developed in the early 1950s for studying structures. Even though they do not necessarily relate directly with morphogenesis, they have certain elements in common which subsequently we will attempt to synthesise.

4.2 Fermi-Pasta-Ulam–Tsingou approach

The Fermi-Pasta-Ulam-Tsingou (FPUT henceforth) problem in non-linear dynamics and vibration theory [Fermi et al., 1965] provides another approach to explaining structural properties and dynamics in economics. Unlike the chemical-mechanical approaches to explaining pattern formation in biological systems, this approach lies entirely within the physical sciences. Unlike also many chaotic systems in which apparent erratic behaviour can result from purely deterministic (non-linear) systems, this problem belongs to a class of problems where instead predictable and regular properties of a structure are themselves unexpected and incredibly hard to explain. A brief introduction to the problem is as follows.[23]

In the early 1950s Enrico Fermi, John Pasta, Stanislaw Ulam and Mary Tsingou studied the problem of thermalisation in mechanical systems in Los Alomos laboratory through one of the earliest examples of experimental mathematics using computers. Consider a string of atoms, suspended in a straight line, and coupled to one another with springs that have small deviations from linearity in how their movements interact with that of the other springs. The problem arises from how energy applied along a given trajectory of possible oscillatory movements – a mode – leads to the system transitioning to a combination of other modes. The expectation was that with nonlinearities introduced in the springs, energy applied to one mode would, over time, end up being shared across all other possible modes. However, while energy was initially shared across different modes as expected, over time the system moved towards complicated dynamics where the vast majority of energy provided returned to the original mode.

Specifically, FPUT investigated a one-dimensional lattice composed of N=64 oscillators (atoms) which are coupled to their nearest neighbour under zero boundary conditions.[24] Let l be the length of a one-dimensional string and the equilibrium positions of

[23]For a historical overview of the problem and later developments, see Weissert [1997] and Galavotti [2008].

[24]It can be instructive to think about a string of atoms (vibrating oscillators) being tied at both ends to a wall.

the oscillators be given as:

$$p_i = ih, \quad i = i, \ldots N-1$$

where h is the lattice spacing. The positions of oscillators at any given time t is specified as $X_i(t) = p_i + x_i(t)$, where $x_i(t)$ is the displacement from equilibrium. The force of attraction between an oscillator and its neighbour in this arrangement is given as $k(\delta + \gamma\delta^2)$, where γ is the strain. Based on the total force acting on the oscillator i (by taking in to account its right and left neighbours), assuming that the mass of all oscillators is m and $\dot{x}(0) = 0$, we have the following laws of motion, with boundary conditions $x_0(t) = x_{N-1}(t) = 0$:

$$m\ddot{x}_i = k(x_{i+1} + x_{i-1} - 2x_i)[1 + \gamma(x_{i+1} - x_{i-1})]$$

When $\gamma = 0$, the interactions are linear and we have a case with $N-2$ coupled oscillators resulting in no thermalisation. When γ is small, it was expected that the system will equilibrate with energy shifting gradually between different modes, leading to thermalisation. To their surprise, they found that not only did thermalisation fail to occur, but the dynamics were much more complicated in terms of the behaviour of the modes, with almost all emergent periodic behaviour difficult to explain. This landmark study led to a large literature on nonlinear dynamics and advanced our understanding of solitons.[25]

There are remarkable similarities between the set up of the problems investigated by Turing and FPUT roughly around the same time. They both consider a string (or a ring) of coupled units, are interested in understanding the process of homogenisation of a system (or lack thereof), where the cumulative effects of initial displacements lead to counterintuitive results. In both cases, the notion of symmetry plays an important role - while pattern formation in Turing's model of morphogenesis is the result of symmetry-breaking, recent research on the FPUT problem and theory of solitons has shed light on the persistence of certain properties in such systems having to do with hidden symmetries (symmetry as understood in light of Emmy Noether's theorem on symmetry).[26] Thus, symmetry analysis and the search for underlying mechanisms associated with preservation or change in systemic properties can potentially lend significant insight for structural analysis.

4.3 Cellular automata, networks and agent-based models

The third approach that we wish to focus on involves agent-based models, particularly those where structural influences play a prominent role. Early developments in the field of agent-based models had explicit structural features in the form of coupling or networks. One of the origins of modern agent-based models lies in the *cellular automata* tradition. John von Neumann and Stanislaw Ulam carried out some foundational work

[25] In particular, through the important contributions by Norman Zabusky and Martin Kruskal (also see Palais [1997]).

[26] To the best of our knowledge, the deeper connections between FPUT, Turing's model of morphogenesis and von Neumann's work on cellular automata was first pointed out by Velupillai [2013].

on cellular automata (viewed as a biophysical computational model) in the 1950s. They were interested in constructing a self-replicating automata so as to understand biological evolution and the self-organizing capabilities of the human brain. Cellular Automata (CA) are discrete mathematical structures (both in space and time) and their evolution is based both on the rules of (local) interaction and the structure of the 'neighbourhood' or network, where the idea of a network is implicit, related to the nature of coupling or physical proximity to nearby cells.[27]

The role of structure in dynamics within the cellular automata tradition can be best understood by examining two early models : (a) John Conway's *Game of Life*, (b) Stephen Wolfram analysis of *elementary cellular automata* [Wolfram, 1983, 2002]. These models have network structures as essential ingredients, and neighbourhood-based decision rules which can be specified as follows:

$$X_{i,t+1} = f(X(N_{i,t})) \tag{7}$$

where $N_{i,t}$ represents the set of *neighbours* of the cell (or agent) i and $X(N_{i,t})$ is the *state* of the neighbours at time t. The neighbours can in principle be defined based on physical or social distance. From the decision rule, the decisions of agent i at time $t+1$ can be seen to depend on the decisions made by the neighbours at or up to time t. In order to relate it to the previous examples, let us consider the simplest case of coupling involving a one-dimensional lattice. The agent has two neighbours (right and left) and needs to make a binary choice (0 or 1). The decision rule of the agent can be described via the mapping

$$X_{i,t+1} = f : \{1,0\} \times \{1,0\} \times \{1,0\} \rightarrow \{1,0\} \tag{8}$$

Wolfram [2002] systematically studied these decision rules for binary strings and generated their time-space patterns. He then classified these decision rules into four classes, which can be seen as representing a hierarchy of complexity. This four class classification links one-dimensional cellular automata, dynamical systems theory, formal language theory and the theory of computation. Expressed from the view point of dynamical systems theory, these four classes are fixed points, limit cycles (periodic cycles), chaos (pseudo randomness, strange attractors), and on the edge of chaos (complex patterns). Thomas Schelling and James Sakoda employed neighbourhood-based decision rules in the form of checker board models to study patterns of segregation in residential choices, where agents' decisions are influenced by their neighbours Sakoda [1971]; Schelling [1971].[28] These models examined how unexpected or undesirable macro-level patterns are plausible from the interactions of several individuals under neighbourhood-based decision rules, within a rational choice theoretic framework. Depending on the distribution of threshold values of tolerance, we can characterise a *tipping point* or *critical point*, at which a morphogenetic change in demographics can occur. Note that the social structure or networks

[27] In a two-dimensional square lattice, the number of cells that constitute a neighbourhood varies depending the definition of distance, such as Chebyshev distance (Moore neighbourhood), Manhattan distance (von Neumann neighbourhood).

[28] Although without explicitly relating them directly to CA or agent-based models.

are exogenously specified in the these models. Another class of models that made use of neighbourhood or lattice structure relate to *Spatial games*, which can be seen as a predecessor to *network games.* They contain a framework akin to the prisoner's dilemma and the game is played by many agents who are distributed across space, with the aim of studying the emergence and evolution of cooperation [Axelrod, 1984; Nowak and May, 1992]. Unlike the early generation models which worked with a *given structure*, later developments in network theory (among others, Watts and Strogatz's *small-world networks*, Barabasi and Albert's *scale-free networks*) made it possible to investigate the endogenous processes underpinning network structure formation.[29]

5 Structure, coupling, diffusion and economic dynamics

> The study of dynamics of the industrial capitalism should not proceed as the solution of a given system subject to exogenous shock. **The economy has been characterized by the fact that it generates not only perpetual motion but one which exhibits continual alteration of its own structure in the pursuit of private profit.** It can be considered either as a single species altering its structure (morphogenesis) or as a selection of new species through competitive survival and/or disappearance. [Goodwin, 1993, p.45, emphasis added]

The paths of evolution of the economic system are determined by the existing structure at any given point in time and the dynamic processes, in turn, alter that very structure. There are also associated questions concerning the stability and complexity of the evolving structures. We believe that this ceaseless process of 'being to becoming' [Prigogine, 1980] ought to be understood not by confining it the narrow strictures of equilibrium, but by instead focusing on the adjustment process of this transformation, mediated by the variety of economic and institutional constraints.

The crucial insights from the approaches described in the previous section that have a potential to form a basis for structural economic dynamics can be synthesised as follows: (a) the role of coupling as a modelling percept for gaining insights into structure; (b) the role of diffusion in studying endogenously generated inhomogeneities; (c) the idea of symmetry-breaking. Although they are interrelated, but there is subtle difference in their roles of the first two. Coupling emphasises the importance of structures that are capable of producing interesting dynamics. On the other hand, diffusion has to do with the functional role of the forces through which changes come about. The other overarching theme in this approach is indispensability of the algorithmic methods, which offer a powerful way to move forward from baseline models with simplifying assumptions.

[29] For a detailed discussion the evolution of agent-based models through the lens of networks, see Chen and Venkatachalam [2017].

We move on to briefly discuss how the two concepts identified have a place in economic investigations.

5.1 Coupling and diffusion

The idea that coupling – interconnections – between different economic units is an important factor in understanding economic structures and dynamics is not entirely new to economic theory. It is natural to think of the economy as comprised of different agents, sectors, regions, and countries that are interdependent, at the least because outputs from one sector are frequently inputs for production in another sector.

In this regard it may be useful to distinguish between two kinds of coupling: static and dynamic. Under static coupling the economy is characterised through interconnections between sectors. This approach has a rich tradition in economics, including the input-output models of Leontief, multi-sectoral models of various types, and Sraffian systems. In the case of dynamic coupling, different countries can be viewed as economies coupled to each other in multiple ways, for example through trade flows, technology, factor flows and financial flows. Variations in the intensity of interdependency between different components (sectors, countries) is expected to a have an influence on the aggregate dynamics and patterns of the economy. Note that interconnections (*structure*) are only one part of the story, and what flows *through* these interconnections (e.g. traded goods, financial capital, factor services) is equally important in determining the final pattern. Given the same intensity of interconnections, we intuitively expect different kinds of flows (say goods vs. financial capital) to give rise to different patterns and outcomes, both due to their varying roles as well as the associated economic and institutional constraints they face within the structures of production and consumption. Overall, analogies between these flows and diffusing agents or form creators, however inadequate, can be instructive. Equally, the role of competition and factors of production (labour and capital) are also amenable to investigation through the lens of diffusion and morphogens.

The key then is to identify the relevant structures and economic forces which can explain observed patterns.[30] Let us first consider Turing's approach to pattern formation and his emphasis on coupling – thereby endowing a structure or geometry – to study interactions between component parts as the source of dynamics. This idea was independently introduced in economics by Goodwin [1947], a theme that resonates in many of his later works. His model of the economy as a coupled system with different sectors investigates the effect of dynamic coupling in the presence of production lags. Some notable works in this strand include Zambelli [2011, 2015] and Lorenz [1987]. Zambelli works with a system of economies, each a (non-linear) oscillator, coupled through trade. Each economy is formulated in terms of a Hicks-Goodwin-type dynamic multiplier-flexible accelerator model, generalized to allow for openness, where each oscillator (economy) is "forced" by the effect of trade flows. Through numerical simulations, he investigates how the dynamics of the system can shift from limit cycles to chaotic attractors, and demonstrates

[30]For the purpose of this paper, we focus on spatial and sectoral inhomogeneities that are endogenously generated via coupling.

the presence of rich dynamics such as *mode-locking* and the *devil's staircase*. Lorenz [1987] instead works with a system of three oscillating sectors, each characterized by a Kaldor (1940)-type nonlinear model. The source of coupling are the interconnections through demand and investment. By interpreting coupling as a source of perturbation on a three-dimensional torus, he invokes the theorem by Newhouse, Ruelle and Takens to demonstrate the possibility of a *strange attractor* and explores this numerically.

The unifying theme across these studies is the role of coupling that leads to a qualitative change in the dynamics of the system. Zambelli [2015] attempts to isolate the effect of coupling on oscillating economies in a lattice explicitly in the FPUT framework and shows that the effects of small perturbations do not die down, thereby permanently altering the nature of oscillations within individual economies. There is also a sizeable literature on diffusion of innovations, technology across sectors and space [Hall, 2004; Rogers, 2010; Zanello et al., 2016]. Their subsequent effect on growth and transformation has been studied in neoclassical and evolutionary frameworks.

5.2 Symmetry-breaking

Symmetry-breaking is the second aspect of the morphogenetic approach. An early exposition of this idea in the context of the social sciences can be found in Allen and Sanglier [1978], who employ a model highlighting the role of economic forces in transforming a homogeneous region into an urban one with different population concentrations. Even though the idea itself may not be new, the connection with symmetry-breaking is what is relevant for our purposes. Theoretical contributions using related ideas to understand trade patterns and *Economic Geography* can be found in Krugman [1991], Krugman and Venables [1995a,b], where transport costs ('iceberg' costs) play a crucial role in explaining how the core and periphery are created as a result of symmetry-breaking. Krugman and Venables make an explicit reference to Turing's work on morphogenesis and their model (the continuous version) is along similar lines to those developed by Turing. Predatory-prey models have been fruitfully employed in economics by Richard Goodwin to explain growth cycles, and have also been used to analyse spatial patterns emerging through diffusion [Aly et al., 2011]. Turing bifurcation is conspicuous by its relative absence in the context of models dealing with economic dynamics. Exceptions to this include Fujita et al. [1999], Akamatsu et al. [2012], de Córdoba and Galiano [2020] and Velupillai [2005, ch. 7].

Notable works that deal with symmetry-breaking in economics within the neoclassical framework are by Matsuyama [1996, 2002, 2004, 2008]. In the context of complimentarity games, diversity is often explained by demonstrating the presence of multiple equilibria, which result from coordination failures between players aiming for better equilibria. Instead, Matsuyama [2002] emphasises symmetry-breaking bifurcation to explain observed economic diversity across space, time and groups. Matsuyama [2004] makes a case that inequality among nations can result endogenously when there is financial market globalization. This is shown in an overlapping-generations setting with credit market imperfections. The initial, symmetric steady state (where countries are equally rich) loses its stability with the globalization of financial markets and moves to an asymmetric steady state where rich and poor countries co-exist. The key idea underpinning this approach is to show how small variations or advantages can build up over time, resulting in persistent patterns. The main ingredients of this class of model are the presence of agglomeration effects (increasing returns) which overpower the effect of (equilibriating) diminishing returns, generating complimentarities through feedback effects. The cumulative effects eventually lead to symmetry-breaking once the steady state loses stability.

Similar to the activators and inhibitors in reaction-diffusion systems, increasing and decreasing returns in these models lead to symmetry-breaking, eventually giving rise to various inhomogeneous patterns. In both cases, it is the difference between the intensities of the two factors which eventually gives rise to symmetry-breaking. Likewise, there is similarity with endogenous models of business cycles in the multiplier-accelerator tradition, in terms of the basic forces at play. The accelerator mechanism can be viewed as the force (though unstable) that propels the system, while the multiplier produces a dampening effect. The relative power of these two mechanisms determines the overall behaviour of the system. This realization led Goodwin to develop his nonlinear model of endogenous economic fluctuations, in which the system remains locally unstable, but globally stable.

6 Dynamic patterns, economic theory and indeterminacy

> Now, after many years ... I sympathise much more with [Schumpeter's] point of view. ... Like Marx he was a student of the morphogenetic nature of capitalism. The economy is not a given structure like von Neumann's model, ... , **it is an organism perpetually altering its own structure, generating new forms**. Unlike most organisms it does not exhibit durable structural stability: it is perhaps best thought of as a kind of hyper-Darwinian, perpetual evolution. [Goodwin, 1989, p.107-108, emphasis addded.]

The main thrust of this article is that the study of capitalistic economic structures can be fruitfully supplemented through the use of morphogenetic frameworks. There are a number of possibilities that emerge from the pathways and connections we have attempted to elucidate, as well as certain caveats.

First, moving from morphogenetic models in their current, mathematically-idealised forms towards forms adapted to enquiries in economics would likely introduce new complexities in the original models, but it is possible that the conceptual innovations needed to tackle these would enrich the morphogenetic literature itself. As Turing notes, (see fn. 31) increasing complexity with the introduction of relevant non-linearities and the study of evolution of inhomogeneous patterns may come at the cost of mathematical tractability and overarching theories. Perhaps this alerts us also to the trade-offs involved in subscribing to a narrow methodology focused on establishing existence-uniqueness of equilibria, optimal trajectories, cycles and so on.

Second, the alternative, i.e., developing rigorous algorithmic methods to investigate structural evolution through a careful investigation and classification of special cases (much in the spirit of Carl Linnaeus), may prove fruitful. This will involve studying transitional paths in the light of economic and institutional constraints. There is also the case of using policy to redistribute resources across different parts of the system – a special feature of economic systems – which can alter growth trajectories. Such counterfactual experiments concerning structure and dynamics can exploit the power of computational frameworks. Third, the crucial issue of the economic system 'perpetually altering its own structure, generating new forms' that Goodwin points to has yet to be convincingly addressed. Increasing product varieties, new technologies, and sectors emerging endogenously as a result of economic dynamics, all need significant unpacking. This points to synergies that ought be exploited between structural and evolutionary perspectives. Fourth, notions of stability from dynamical systems theory are often applied uncritically in economics. It may be worthwhile to develop instead a notion of stability that is tailored to economic problems. To give an example, notions like structural stability may be too restrictive, and other notions of stability that focus on aspects of *relative invariance* within a structure may be useful [Scazzieri, 2012]. To this end, a comprehensive theory of *intermediate stable forms* and its integration with structural evolution remains wanting.

That noted, the mere existence of a mathematical structure capable of generating interesting dynamics does not in itself warrant an application to economic models, especially if this involves untenable assumptions to suit the requirements of the mathematical approach. Clearly, the models discussed all have limitations as well as simplifying assumptions.[31] Yet they also share several interesting features vis-à-vis defining and categorising economic structures and their dynamics, and the onus is on the economic theorist to demonstrate the suitability of a model for studying economic processes *as they exist*. Equally, an outright rejection of these cross-disciplinary insights is also unlikely to be tenable – clearly, several significant advancements in scientific theory have originated from insightful analogies and interdisciplinary perspectives, and economic theory is no exception.[32]

[31] For instance, Turing [1952, p.72, emphasis added] acknowledges that perhaps a more relevant case in morphogenesis may be the one in which organisms are seen to develop from pattern to pattern and not from homogeneity to pattern (for some recent progress on tackling this issue, see Krause et al. [2020]). Similarly, the role of linear specification and uniformity assumption concerning the individual cells can be seen as restrictive.

> The difficulties are, however, such that one cannot hope to have any very embracing theory of such processes, beyond the statement of the equations. It might be possible, however, to treat a few particular cases in detail with the aid of a digital computer. . . . The essential disadvantage of the method is that one only gets results for particular cases. **But this disadvantage is probably of comparatively little importance.**

[32] Famous metaphors range from the invisible hand, rocking horses, pendulums, oscillators, sunspots, predator and prey, prisoner's dilemma, trembling hands, utility computers, turnpikes, cob-webs, billiards players, random walks - to name just a few.

We conclude with some remarks on the epistemological dimensions of predictability, and in particular the quest to predict and characterise novelty within a system. Morphogenetic frameworks allow for the emergence of potential surprises even in non-stochastic systems. They allow us to pose questions that are directly relevant for policy: Given a state (pattern) X, can we decide, *in general*, whether the system will reach an attractor (pattern) Y? By decidability we are referring here to problems that can be resolved by means of an algorithmic procedure. The question can also be posed in recursion-theoretic mode as a problem of *reachability*, and it can be proved that in general, reachability is algorithmically undecidable for nonlinear systems. If a structure is 'sufficiently complex', such undecidabilities concerning (disequilibrium) dynamics may be even more pervasive [Velupillai, 2010]. In this sense, epistemological incompleteness will always exist, but we believe that this is to be embraced: the general undecidability concerning morphogenesis means simply that we need to examine scenarios through simulations in order to understand actual future paths of evolution.

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