Sraffa, Money and Distribution

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Abstract

Sraffa's early work on monetary economics, his contributions to the theory of capital and his critique of neoclassical tenets are often seen as disjoint contributions. In contrast, we suggest that Sraffa's contributions have to be viewed as a coherent whole and offer a classification of his scholarly contributions. We point to the unifying thread between his early and later work, which concerns the insufficiency of economic mechanisms or market forces to exclusively determine values (prices, profit rates and wage rates). Their simultaneous determination and consequently the overall indeterminacy of the system are important. We outline the motivations for incorporating money (in the form of credit and debt) inside the traditional Sraffian schemes, to expand the original system and harness its potential. We believe that there is both a need and scope for incorporating deferred means of payments, which are essential for the functioning of any evolved economic system.

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1 Introduction

Sraffa's contributions to economics can be broadly divided into two parts, which may at first seem disconnected. The first part of his research was dominated by themes that are closely related with monetary economics and banking theory (Sraffa, 1920, 1922a, 1932a). The second concerns themes related to the production, measurement and distribution of physical surplus in an economy (Sraffa, 1925, 1926, 1951, 1960). The second part concerning value and distribution has received quite a lot of (well-deserved) attention from scholars, while his early writings on monetary themes have received comparatively little attention. The connection between these two parts and how one may have influenced the other have often been overlooked.

The absence of money in Sraffa's magnum opus, *Production of Commodities by Means of Commodities*(Sraffa, 1960, from now on PCMC), has led many to conclude that money is not an important element in his theoretical system. But Sraffa has been explicit in stating that his system is indeterminate. In other words, it is not possible to simultaneously determine the self-replacing prices, wage rate and profit rate, with the knowledge of the methods of production alone (see the Appendix).

Bharadwaj (1963, p.1450) highlights this point explicitly:

> Distribution in Sraffa's system is not endogenously generated through production relations. [This is a significant departure from the widely prevalent practice of obtaining distributive shares from the production function applying the marginal method]. In conjunction with prices, production relations determine only the net surplus that is to be distributed. [Rather prices and the rate of surplus are determined **simultaneously** given the production relations]. No theory of distribution is offered in the book. His wage rate is a variable that could be considered to vary with the same ease as the profit rate. Wage is assumed to be paid post factum, so that profits and wages are surplus sharing entities. (emphasis added, text within paranthesis are footnotes in the original version)

Sraffa takes one of the prices (i.e. the uniform rate of profits) to be an independent (i.e. exogenous) variable, there by making his system determinate. It is important to note that this assumption concerning choice of the uniform rate of profits is in itself not crucial. One could equally work with choosing wage rate or a commodity price as the independent variable. Hence, the important point concerns the indeterminacy in the Sraffian system, which implies that there is a crucial role for forces outside the economic system in the determination of self-replacing prices.

Sraffa remarks that the money rates of interest, which may be determined from outside the system of production, might influence the rate of profits. As we show below, it would be incorrect to interpret Sraffa as suggesting that the system would be determined when the money interest rates are given. Quite the contrary! Sraffa infact makes a reference to the money rates of interest to reinforce the idea that economic forces alone are not sufficient to determine exchange-values and distribution.[1]

Therefore, when one attempts to reconstruct economic theory with Sraffian schemes and Sraffa's work as the starting point, it would be essential to consider institutional factors and the banking/financial sector. More generally, one ought to consider the power

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[1]In the correspondence with Garegnani, Sraffa makes this point quite forcefully, see below p. 10

relations that underpin production activities in an economy that could decisively shape the distribution of physical surplus.

The money rates of interest is one of the tools that can be used to control financial costs and hence profits of production units. A selective use of financial funds favouring some sectors with respect to others, would obviously influence the exchange values as well. However, it is not the only way in which the power of certain groups may influence the exchange values and hence distribution. The recognition that the system is indeterminate and allocations of funds may be controlled by powerful groups opens up space for investigating a monetary explanation of distribution. Motivated by this, we argue that money (seen as deferred means of payment), offers a fruitful (and necessary) generalisation of the Sraffian schemes.

In section 2, we provide a brief overview of Sraffa's contributions. We argue that money was very much a part of his research agenda. We offer a classification of his contributions and argue that they have to be viewed as a coherent whole. In section 3, we focus on the explicit and implict role of money and distribution in PCMC and, more broadly, in Sraffa's work. We idenitfy and emphasise a common theme in his work that concerns the insufficiency of the natural forces of the market in determining the prices, rates of profit, wages and the distribution. In section 4, we outline the motivations for generalising Sraffian schemes by incorporating money, viewed as a deferred means of payment. Appendix A provides a formal treatment of Sraffian schemes for the case of single product industries extended to the general case of non-uniform rates of profits. Appendix B provides a concise outline of how credit and debt can be incorporated inside the traditional Sraffian framework.

2   An overview of Sraffa's contributions

Sraffa's first scholarly publication was his graduate thesis in 1920, titled *L'inflazione monetaria in Italia durante e dopo la guerra* ('Monetary Inflation in Italy during and after the war', Sraffa (1920)) written under the supervision of Luigi Einaudi. In this, Sraffa focused on the themes related to money and monetary policy in the Italian context of that time. It investigated the causes of inflation, analysed issues concerning stabilisation of domestic prices, exchange rates and distributional aspects. This work, as argued in Panico (1988), is quite applied in character and bears many similarities with A *Tract on Monetary Reform* by Keynes.

Apart from his lucid and incisive political commentaries in *L'Ordine Nuovo* (Sraffa, 1921), his next significant scholarly publication was also centred around the Italian monetary affairs. Keynes, who was then the editor of *Manchester Guardian Supplement on Reconstruction in Europe* had commissioned Sraffa to write an article for the magazine. This led to two articles - one in the *Manchester Guardian* and other was published in the *Economic Journal*. His analysis of the case of Banca Italiana di Sconto which went bankrupt by the end of 1921 was particularly insightful.

These articles (Sraffa, 1922a,b) focused on the crisis of the Italian banking sector. They showcase his remarkable familiarity and understanding concerning the operations of the banking sector, the regulatory framework, its relationship with the industries, the role of interest groups and the myriad of issues they face.[2] Thus, characterisations of

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[2] It is also important to note in this context that Sraffa has spent time learning the practical workings of the wheels of banking from the inside as an apprentice at the provincial bank *Banca di Legnano e*

Sraffa's body of work merely as being an exercise in theory with no practical relevance cannot be justified. This is evident even from a cursory look at the explicit themes that characterise his contributions on money, even without delving into the implicit practical, empirical and policy implications of his theoretical work.

Sraffa (1925) put forward an important critique of Marshallian theory of firm and its relation to the equilibrium position of an industry. This was published in the Italian journal *Annali di economia*, titled *Sulle relazioni fra costo quantitá prodotta*[3]. This contribution, which systematically analysed the relationship between cost and quantity, is to be seen in the context of a raging debate between different scholars during the 1920s on the 'law of returns' in the Marshallian theoretical apparatus. A summary of this article was published in the *Economic Journal* (Sraffa, 1926).

Sraffa's review of Hayek's *Prices and Production* (Hayek, 1931a, 1932a; Sraffa, 1932a,b) published in the *Economic Journal* deconstructs a theoretical system put forward by Hayek and highlights some inconsistencies in Hayek's framework. Sraffa's review also highlights his nuanced understanding of money, monetary theory and stabilisation policy.[4] In particular, his views on the nature of monetary interest rate, attempts to compare it to the 'natural' rate of interest, and the impossibility of having a unique equilibrium interest rate were important insights.

Sraffa undertook the task of editing the collected works and correspondences of David Ricardo in the 1930s. This monumental, meticulous work took well over two decades and was published the early 1950s. In the first volume - *On the Principles of Political Economy and Taxation* - Sraffa offered an impressive introduction to Ricardo's works and resurrected the classical surplus approach to political economy, where circular flow takes a centre stage. Many scholars regard his introduction as par excellence in offering a balanced, informative and interpretive judgement on a great scholar's body of work.

Sraffa's arguably best known work, *Production of Commodities by Means of Commodities*, was published in 1960. In this, Sraffa builds up a system following the classical tradition. Given the methods of production and taking one of the distributive variables as being exogenously given, Sraffa resolves some important issues concerning value and distribution. In particular, he has shown that values (i.e., self-replacing prices) and distribution cannot be computed independently of each other. There is a unique association between prices and the distribution of the physical surplus. Therefore, given a distribution, we can calculate the prices or vice-versa. This was and is a devastating result for (Neo-) Walrasian general equilibrium and Austrian value theory and for any attempt to measure aggregate capital and aggregate national product independent of the distribution.[5]

In Sraffa's framework, distribution becomes the key aspect in determining the rela-

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Busto Arsizio post his graduation.

[3]Sraffa's Italian translation of *A Tract on Monetary Reform* was also published in 1925. Sraffa also published brief book reviews in the *Giornale degli Economisti* that concerned monetary and banking issues.

[4]Although the title of Hayek's book is *Prices and Production*, it is really about a monetary theory of the business cycle. The title of Sraffa's review in fact is *Dr. Hayek on Money and Capital*. One of Sraffa's points was that a disequilibrium scenario requires the use of deferred means of payments - i.e., money or credit-debt contracts.

[5]This point made by Sraffa is often misunderstood or misinterpreted. Zambelli (2018b) takes the numerical values used by Sraffa and demonstrates that the value of capital and the industry capital/output ratios are all functions of distribution (prices). The values of aggregate capital, or the value of capital per industry do not remain constant as the distribution changes. Furthermore, their changes are not monotonic with respect to changes in distribution.

tive prices. It is a penetrating (prelude) critique of neoclassical theory and its marginal foundations. This slender volume breathed a new life into the classical, surplus based approach and offered it as a credible alternative to the marginal approach. Finally, there is also his important interjection in the Corfu conference on Capital theory, which in our opinion provides us a hint to find a persuasive way to interpret his body of work.

2.1 Sraffa's contributions: a coherent whole

Sraffa's early work, as we have outlined previously, is highly applied in character. It focuses on monetary and banking issues. Around the same time, he dealt with some thorny issues in pure theory. The later part of his work focused mostly on theoretical issues concerning value, distribution and capital, with relatively less applied elements. This has often led scholars to conclude that his work on the two areas - money and capital, for the convenience of exposition - as disconnected, distinct spheres of Sraffa's scholarly contributions.

Instead, we argue that his early work on money, his critique of marginal theory and capital are not isolated exercises. Following Panico (1988b), we claim that Sraffa's research endeavours in these two parts are coherent with respect to a focused research agenda.[6] First, it is important to bear in mind that Sraffa's research agenda had been set fairly concretely during the 1920s and early 1930s. We now know that essential aspects of his theoretical system were already in place by then. In fact, his subsequent research work was mainly about furthering this research agenda, which only saw marginally modifications over time. Second, from his early writings on money, it is evident that he had a concrete stance about the role of power, class, interest groups and non-market forces in the determination of the money rates of interest. There is no evidence that we can find to believe that his stance had changed in any substantial manner throughout his academic life. Third, we argue that this position can be seen to later manifest itself in the form of Sraffa having an *exogenous* distribution. This exogenous distribution variable (profit or wage rate), as argued by many scholars (Bharadwaj, 1963), is a crucial element in determining the relative prices in PCMC.

Even if one accepts the lack of complete disconnect between Sraffa's early and later writings, we are still left wanting for a coherent narrative to understand his body of work. To this end, we propose that Sraffa's research may be characterised as one that is geared towards achieving precision in theory. In order to make this point clear, we construct a spectrum in which contributions range from being applied in nature on one end to being purely theoretical on the other end (see Fig.1).

- Works closer to the applied end of the spectrum take existing theory largely as a given and explore innovative applications and interpretations, without drastically

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[6]In his paper discussing Sraffa's early work Sraffa (1920, 1922a) on money and banking, (Panico, 1988b, p.7) underlines that:

> Sraffa started his research working on applied monetary and banking problems, making some relevant contributions to the area. The theoretical problems that made his fame as an outstanding theoretician came to dominate his interests only at a later stage.

Panico also argues that:

> . . . there is a close link between [Sraffa's] earlier writings on applied problems and his later theoretical works, a link which clarifies the origins of his theoretical interests, showing the error of considering his contributions as 'merely abstract exercise in pure theory'.

challenging or reworking the theoretical premises.

- Contributions closer to the right-hand side of the spectrum, on the other hand, constitute fundamental contributions that challenge, construct and clarify core theoretical categories.

The former, given their applied nature, necessarily deal with and even rely on imprecise measurements. No such imprecision is allowed when it comes to purely theoretical concepts and definitions involving economic measures. This attitude is well encapsulated in his intervention at the 1958 conference of the International Economic Association at Corfu on the *'Theory of Capital'*.

> "[O]ne should emphasize the distinction between two types of measurement. First, there was the one in which the statisticians were mainly interested. Second there was measurement in theory. The statisticians' measures were only approximate and provided a suitable field for work in solving index number problems. **The theoretical measures required absolute precision. Any imperfections in these theoretical measures were not merely upsetting, but knocked down the whole theoretical basis**. [...] The work of J. B. Clark, Böhm-Bawerk and others was intended to produce pure definitions of capital, as required by their theories, not as a guide to actual measurement. If we found contradictions, then these pointed to defects in the theory, and an inability to define measures of capital accurately. It was on this - the chief failing of capital theory - that we should concentrate, rather than on problems of measurement." – Piero Sraffa (Lutz and Hague, 1961, pp. 305-306, emphasis added).

We can place Sraffa's scholarly contributions to the two different spheres (money and capital) on the above mentioned spectrum in Fig.1.

First, on Sraffa's contributions relating to monetary matters:

a) On the left hand side of the segment, we place his work on the causes of inflation and issues concerning monetary stabilisation; institutional characteristics and regulatory framework that characterised the Italian banking system in the early 1920s and their consequences; his analysis of the relationship between industrial and banking sectors; the role of interest groups (Sraffa, 1920, 1922a,b).[7]

b) On the right hand side (theoretical extreme), we place his review of Hayek's 'Prices and Production', where his critique strives to highlight and remove imprecisions from pure theory that concern the role of money in business cycles.[8]

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[7]In his thesis, for instance, he takes the quantity theory of money as a given and applied it to monetary issues of his time (Panico, 1998, p.10).

[8]His critique of Hayek's book was made alongside Keynes' own critique of Hayek (Keynes, 1931; Hayek, 1931b, 1932b; Sraffa, 1932a,b). There is evidence of Sraffa's involvement in the corrections of 'A Treatise on Money' Keynes (1930) as well. It is worth noting that Sraffa translated (Keynes, 1923, 'A Tract on Monetary Reform') into Italian and was involved in the discussions surrounding the writing of Keynes' 'General Theory'. In this context, it is also important to bear in mind that Chapter 17 of the *General Theory*, (Keynes, 1936, Ch: The Essential Properties of Interest and Money) may be interpreted as addressing the relation between interest and money using some of the arguments put forward by Sraffa in his review of Hayek. In fact, Keynes does acknowledge that the idea of the own rate of interest was taken from Sraffa's critique (Sraffa, 1932a) of Hayek's 'Prices and Production' (Keynes, 1936, p. 223, fn.1). All these attest to the interpretation that money and monetary issues were not in anyway insignificant in Sraffa's academic interests.
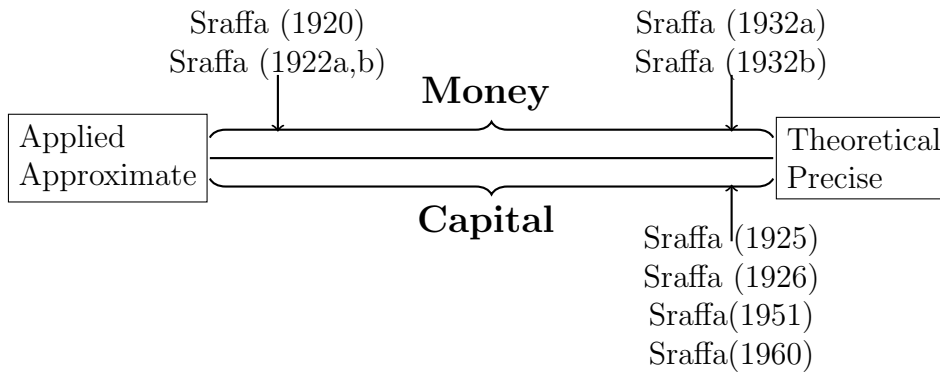
Figure 1: A classification of Sraffa's contributions

Second, moving on to Sraffa's contributions pertaining to value, distribution and measurement of capital (non-monetary issues, loosely speaking, which we characterise under capital for the ease of exposition): all of them invariably fall on the right-hand extreme of the spectrum - involving theoretical contributions which require definitions and measurements with utmost precision. Thus, in the right end of the spectrum, we have the problems of:

- measurement of capital and returns at the firm level (Sraffa, 1925, 1926)

- value and the measurement of capital at the aggregate level (Sraffa, 1951, 1960)

In each of these cases, Sraffa is uncompromising in his theoretical rigour, be it in fleshing out inconsistencies in Marshallian economics, clarifying the conceptual issues in Ricardo's theoretical apparatus or constructing a theoretical system of his own. It certainly won't be an exaggeration to claim that precision in pure theory has been one of the major concerns of Sraffa throughout his academic life. For instance, other than the works mentioned above, his interventions in the famous 'Symposium on increasing returns and the representative firm' (Robertson et al., 1930, pp. 89-92, 93) bear evidence to his penchant for theoretical precision. Equally, there are also several instances in his archives and correspondences that further support this. Sraffa was critical of the use of marginal productivity theory to determine aggregate wage shares by Hicks in his *Theory of Wages*. In his correspondence with Hicks (Sraffa papers, C133, undated), Sraffa objects to certain assumptions concerning the elasticity of substitution. These astute remarks may be seen as an anticipation of capital theory debates of the 1960s.

Although we have placed Sraffa's works exclusively on one of the two spheres, we need to exercise caution while interpreting such a characterisation. For example, *PCMC* (Sraffa, 1960) certainly has elements which are clearly related with money and credit. This can be seen from the importance that he places, explicitly or implicitly, on the relevance of money interest rates in influencing the exogenous distributive variable. One may argue that these reference points found in PCMC are simply 'marginal'. They may be so for a *critique* (or a prelude to a critique) of certain kinds of economic theory. However, they are, at least in our view, central starting points for a rigorous generalisation of Sraffian schemes. We discuss more about money and distribution in Sraffa in the next section.

3 Sraffa, PCMC, money and distribution

We have outlined earlier that money was very much a part of Sraffa's research interest right from his early days as an economist. The lack of detailed discussion of money in PCMC has led many to believe that money perhaps was not important in his theoretical system or that there is no place for it. We argue that money (seen as credit and debt), far from being irrelevant, offers a fruitful generalisation of the Sraffian schemes. Second, even within the context PCMC, there are two important points where the money rates of interest become particularly relevant.

3.1 Importance of money rates of interest: explicit reference

The explicit point concerning the importance of money rates of interest in PCMC is in the well known quotation:

> The rate of profits [. . .] is accordingly susceptible of being determined from outside the system of production, in particular by the level of the money rates of interest (PCMC, p.33).

The relationship between the rate of profits and the money rates of interest is one of the central themes of economic theory. In particular, a crucial issue is whether there exists a (unique) *natural* rate of profits which is related to the productivity of physical capital. Additionally, whether money rates of interest converge towards a natural rate of profits and whether the sectoral rates of profits converge towards a unique money rate of interest is also relevant. Fisher (1906, 1907, 1911) and Wicksell (1936) have made important contributions to the debate centred on these issues.

If the money rates of interest determine the rate of profits of commodities, and this rate of profits, in turn, determines the distribution of the physical surplus, it reasonably follows that the very forces that determine the money rates of interest implicitly determine the distribution as well.[9]

If the money rates of interest do indeed regulate the rate of profits, then it could be argued that specifying this functional relationship in some unique manner can help ameliorate the issue of exogeneity, making the system determinate. However, Sraffa does not find merit in this. That the money rates of interest are important factors (but not the only one) for the determination of the distribution of the surplus is stated in a response to Garegnani (Sraffa Papers D3_12_111_144-146), who was seeking clarifications with respect to the above quotation.

In 1962, Garegnani was preparing a review of PCMC to be published in the Italian journal *Moneta e Credito*. In a correspondance with Sraffa, Garegnani enquired about the relation between the profit rate and the money rates of interest. Garegnani seemed to be in doubt on whether he should address the issue of the profit rate as determined by the money rates of interest.

> [...] you opt for the rate of profit "determined from outside the system . . . in particular by the level of the money rates of interest" [. . .] I am reluctant to venture, even if only with hints, on the terrain of a theory of profit determined

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[9]This was an important point already put forward in Sraffa (1922a) and subsequently in his debate with Hayek (Sraffa, 1932a). Chapter 17 of Keynes (1936) may be interpreted to be addressing this problem using arguments put forward by Sraffa.

by the money rates of interest, with the risk of attributing to you opinions that you do not have.

This for the review. The question however interests me a lot [...] Expressed in very rough terms the way I see the problem is as follows. From Ricardo to Wicksell the idea was that **the rate of interest** would **adapt** to the **rate of profit** obtained from production.

The **justification for that theory** lies, it seems to me, in the idea of a **demand for loans with almost infinite elasticity** (given the time required to formulate the investment plans) around the levels of interest corresponding to the rate of profit obtained in production. If such an elasticity in fact would exist, the **market interest rate** should always tend to its **"natural"** level (any attempt by the monetary authority to fix a different rate by creating or destroying credit would fail, then, because the resulting inflation process or deflation would force to return to the "natural" rate). [Sraffa Papers, D3/12/144-145, our translation, emphasis added].[10]

To this question, Sraffa replies in his letter as below:

From the point of view of your research it seems to me that it is difficult to unify the views of Ricardo and Wicksell, even though they both make the **rate of interest** depend on the rate of profit. But as for the **"rate of profit"** they only have the words in common: for the former it is the excess above the wages given, for the latter the marginal product of capital! How can you put them together? I would like to know the sense in which it can be said that " the justification for that theory" lies in the' 'demand for loans with almost infinite elasticity at an interest rate equal to the rate of profit". **Justification my foot**! I call it an " identical repetition in difficult words", a serious tautology.

I do not see the difficulty of determining the rate of profit by means of a rate of controlled or conventional interest, **provided that the rate of profit is not assumed to be determined by external unavoidable circumstances**. [Sraffa Papers, D3/12/111/150, our translation, emphasis added].[11]

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[10]Text in the original:

[. . .] Lei opta [. . .] per il saggio del profitto "determinato da influenze esterne del sistema . . . e particolarmente dal livello dei tassi monetari dell'interesse" [...] sono restio ad avventurarmi, anche solo per accenni, sul terreno di una teoria del profitto determinato dai saggi monetari dell'interesse, col rischio di attribuire a lei opinioni che non ha.

Questo per la recensione. La questione però mi interessa molto [...] Espresso in termini molto rozzi il modo con cui vedo il problema è questo. Da Ricardo a Wicksell l'idea è stata che **il saggio di interesse** si **adeguass**e al **saggio del profitto** ottenuto nella produzione.

La **giustificazione di quella teoria** sta, mi sembra, nell'idea di una **domanda di prestiti di elasticità pressoché infinita** (dato il tempo richiesto per formulare i piani di investimento) intorno ai livelli dell'interesse corrispondenti al saggio di profitto ottenuto nella produzione. Se infatti tale elasticità esistesse, il **saggio di interesse di mercato** dovrebbe sempre tendere al suo livello **"naturale"** (l'eventuale tentative della autorità monetarie di fissare un diverso saggio creando o distruggendo credito fallirebbe, allora, perché il risultante processo di inflazione o deflazione costringerebbe a ritornare al saggio "naturale") [Sraffa Papers, D3/12/111/150, emphasis added].

[11]Text in the original:

But Sraffa goes on to stress his disapproval of any mechanical theory of determining distribution in the economic system.

> ...I am convinced that the maintenance of **the rate of interest** by the bank or the stock exchange has played its part in determining the distribution of income among the social classes: because it is an **obligatory passage** for those who give and those who borrow loans.

> ...I did not mean anything very binding, and in general I only wanted to send out some signals **to avoid anyone to think** that the system [of PCMC] is presented [by me] as a "foundation" for a theory of relative supplies of capital and labour! It is the <u>negation</u> that seems important to me: as to the <u>affirmative</u> I have no intention of putting forward yet **another mechanical theory** which, in one form or another, reinforces the idea that distribution is determined by natural, or technical, or perhaps even accidental circumstances, but such as to render **futile** whatever **action**, from one side or the other, aimed to **modify the distribution**. (Sraffa Papers, D3/12/111/149, our translation, in bold emphasis added)[12]

Clearly, Sraffa views banks and the stock market (which are relevant for setting of the money rates of interest) as being important in the determination of distribution. But he makes a very subtle point. In PCMC, Sraffa does show that the determination of prices, and hence distribution, cannot be determined *exclusively* within the economic system. In his system, there is *at least* one degree of freedom and he picks the rate of profits to be the variable which is determined from outside the economic system (see Appendix A). Once the rate of profits is given, the distribution of the surplus is simultaneously determined. This is the *negation* that Sraffa seems to emphasise, i.e. the fact that the

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> Dal punto di vista dei suoi studi mi pare che sia difficile unificare i punti di vista di Ricardo e Wicksell, per quanto entrambi facciano **dipendere** il **saggio dell'interesse** dal **saggio del profitto**. Ma quanto al "saggio del profitto" hanno in comune solo il nome: per il primo è l'eccedente al si sopra del salario dato, per il secondo il prodotto marginale del capitale! Come si fa a metterli in pariglia? Io vorrei sapere in che senso si possa dire che "la giustificazione di quella teoria" sta nella "domanda di prestiti ad elasticità infinita a saggio di interesse uguale al saggio di profitto". **Giustificazione un corno!** Io la chiamo "ripetizione identica in parole difficili", grave tautologia.

> Non vedo la difficoltà della determinazione del saggio del profitto mediante un saggio dell'interesse controllato o convenzionale, a **condizione che non si presupponga il saggio del profitto determinato da circostanze ineluttabili esterne**. [Sraffa Papers, D3/12/111/150, our translation, emphasis added]

[12]The text in Italian:

> ...sono convinto che il mantenimento del **saggio d'interesse** da parte della banca o della borsa abbia avuto la sua parte nel determinare la distribuzione del reddito fra le classi sociali: perché è un **passaggio obbligato** per chi dà e per chi prende a prestito.

> [...] io non ho inteso dir niente di molto impegnativo, e in generale ho solo voluto metter fuori qualche segnale **per evitare che si creda** che il sistema viene presentato come "fondamenta" per una teoria delle offerte relative di capitale e lavoro! E' la <u>negazione</u> che mi sembra importante: quanto alla <u>affermativa</u> non ho nessuna intenzione di mettere avanti **un'altra teoria meccanica** che, in una forma o nell'altra, ribadisca l'idea che la distribuzione sia determinata da circostanze naturali, o tecniche, o magari accidentali ma comunque tali da rendere **futile** qualsiasi **azione**, da una parte o dall'altra, per **modificarla** (Sraffa Papers, D3/12/111/149, in bold emphasis added).

system as a whole is indeterminate (see A.6). But, on the contrary, suppose that there was a theory that would *mechanically* explain the determination of the rate of profits through the money rates of interest. This, in turn, would make the system determinate by the mechanical forces that shape the functioning of the banking system or the stock market. Even in such a case, Sraffa seems to point out that any attempt to modify the distribution would be futile.

The point here is that the allocation of new loans could depend partly on the power structure and decisions of banks that provide the *necessary financial backing* to powerful companies that may be controlled by a few individuals. Obviously, strictly mechanical forces of the economic system are not adequate to exactly capture such tendencies. This seems to be the point that Sraffa is making when he writes "*I did not mean anything very binding (*impegnativo*), and in general I only wanted to send out some signals to avoid anyone to think that the system [of PCMC] is presented [by me] as a "foundation" for a theory of relative supplies of capital and labour!*"[13]

Sraffa seems to take a similar position as early as 1922 Sraffa, where he writes:

> The large industries are stimulated on their part to make themselves independent by acquiring control of a bank so as to obtain from it, without undergoing heavy impositions, **the necessary financial backing**. As a result of this opposition, however, it cannot be said, generally speaking, that either of the two opposite tendencies has the absolute upper hand over the other. The general tendency seems to be towards the elimination of this opposition by the formation of large "groups" of companies of the most varied kinds concentrated round one or more banks, mutually related by the exchange of shares and by the appointment of Directors common to them. Within these "groups" the various interests are all equally **subject to the interests of a few individuals** who control the whole group, possessing on their own only a very few shares of the various companies. Very little is known and very little can be generalised about these groups, on account of the undetermined state of their structure, of their **unofficial character**, of the variety of the various groups, and of the continual shifting of the elements which compose them. What the public knows and feels-not only when disasters take place, fatal to the existence to some of them, or when hostilities break out between one group and another-is **the enormous financial and political power which they have and the frequent use they make of it to influence both the foreign and home policy of the Government in favour of their own interests**. Each group keeps several press organs which support its policy, and some of the accusations made against certain Ministries of being actuated by the interests not of a class, but of private concerns, and of favouring one financial group against another, have no doubt a basis of truth (Sraffa, 1922a, p.196, emphasis added).

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[13]If our interpretation is correct, it is puzzling that many Sraffians have placed a strong emphasis on the mechanics of the economic determination of the long term uniform profit rate.

3.2 Implicit influence of the money rate of interest on the measurement of capital and distribution

The implicit influence of the money rates of interest in PCMC relates to the critique that Sraffa develops about the possibility of measuring aggregate capital. In particular, the use of the Austrian notion of 'roundaboutness of capital' or the 'period of production'. The interest rate in the Austrian framework has an influence on the roundaboutness of capital or the length of production processes (Samuelson, 1966, pp. 568-569).

Sraffa shows in PCMC that the length of 'the period of production' (Ch.VI 'Reduction to dated quantites of labour') cannot be independent of distribution or the rate of profits. The view put forward by authors like Jevons and Böhm-Bawerk was that as the money rate of interest falls, the 'period of production' (roundaboutness of capital) would increase[14].

Sraffa writes:

> (The reduction to dated labour terms has some bearing on the attempts that have been made to find in the **'period of production'** an independent measure of the quantity of capital which could be used, without arguing in a circle, for the determination of prices and of the shares in distribution. [What demonstrated in this book] seems to be **conclusive** in showing the impossibility of aggregating the **'periods'** belonging to the several quantities of labour into a single magnitude which could be regarded as representing the quantity of capital. The reversal in the direction of the movement of relative prices, [...], cannot be reconciled with **any notion** of capital as a measurable quantity **independent** of **distribution and prices**) - PCMC, p. 38, emphasis added.

The critique that Sraffa advances has to do with the lack of theoretical foundation concerning the monotonic inverse relation between the 'period of production' (i.e. capital) and the interest rate.

Sraffa's intervention at the Corfu conference is related to the discussion of Hicks' article, titled *The Measurement of Capital in Relation to the Measurement of other Economic Aggregates* (Hicks, 1961). The concluding phrase in this article is: '*The marginal productivity of capital is the marginal productivity of roundaboutness [period of production], after all.*' (Hicks, 1961, p.31).

It is here that Hicks' economic theory clashes directly against Sraffa's critique. Sraffa was in fact correct that a monotonic relationship between the value of capital (period of production) and the rate of profit cannot be established unabiguously.[15] In our parallel work incorporating money inside the Sraffian schemes, we show that a monotonic relation between value of capital and the monetary interest rate is seldom observed.

We have so far drawn attention to Sraffa's view on the cruicial role of distribution, money rates of interest and his explicit dismissal to engage in building a mechanical theory, where distribution is determined by the natural forces of the market. It is also instructive to contrast Hicks' view and Sraffa's view on the matter.

One of Sraffa's objectives is to determine the

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[14]On the notion of the *period of production* and on Sraffa's personal notes on it, see Sinha (2016, Chap. 5, pp. 111–151). On Sraffa's critique of the period of production as being independent of the rate of profits, see PCMC, Chapter 6, and a rather harsh comment of Sraffa (1962) on the review Harrod (1961) of PCMC.

[15]See also Zambelli (2018a) for an empirical demonstration of this point.

> . . . *set of exchange-values which <u>if adopted</u> by the market restores the original distribution of the products and makes it possible for the process to be <u>repeated</u>;* (PCMC, p.3, emphasis added)

so that the system is in a *self-replacing state.*

Sraffian self-replacing prices are market clearing prices. Sraffian schemes can be interpreted as budget constraints of the overall economic system. It is a system where producers have to exchange their products (endowments) at the end of the production cycle. They do so in order to buy the means of production neccessary for production to be replicated during the next production cycle.[16]

Value and Capital (Hicks, 1939) is recognized as a foundation of Neo-Walrasian General Equilibrium. There are several similarities between the production structure that one observes in Hicks (1939) and in Sraffa's PCMC. Hicks separates the production cycle (the week) and the market day (Monday) in the same way in which Sraffa separates the production cycle (the year) with the annual market day (occurring after the harvest). Hicks also points to his discussions with Sraffa in the *Preface*: "I have had some very useful criticism from Mr. Sraffa" (Hicks, 1939, p.vi).[17] In this book, Hicks also aimed at providing firm foundations to the theory of monetary interest rates:

> [. . .] it is evident that any treatment which pretends to deal with the economic system as a whole [. . .] cannot possibly regard the rate of interest in isolation. It is a price, like other prices, and must be determined with them as part of a mutually interdependent system. The problem is not one of determining the rate of interest *in vacuo*, but is really the general problem of price–determination in an economy where borrowing and lending are practised, and in which the rate of interest is therefore a constituent part of the general price-system (Hicks, 1939, Ch.XII, *The Determination of the Rate of Interest*, p.154).

When we consider Sraffa's response to Garegnani's query (see above p.10), in conjunction with the content found in PCMC, this quote is quite illuminating. At first reading, it seems that Hicks and Sraffa are saying the same thing. But there is a very crucial and fundamental difference.

Sraffa's considers that the rate of profits and/or the money rates of interest may not be determined mechanically by the working of economic forces alone. Therefore, from his standpoint, one can claim that there is a scope for attempts to determine the distribution in a society through attempts to change the power structure or through policy.

On the contrary, in Hicks's world, any attempts to change the distribution of the surplus produced would be *futile* because of the very mechanics of the working of the

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[16]Hahn (1982), Hahn and Petri (2002), Fratini (2018), Garegnani (2000), Mandler (1999b,a, 2002b,a, 2005), Negishi (2014, p.7), Parrinello (2008), Schefold (2005, 2008), Sinha and Dupertuis (2009) are examples where Sraffian schemes are studied in relation to Walrasian (intertemporal) general equilibrium. In these contributions there is a clear acknowledgment that Sraffian schemes are equivalent to the aggregate budget constraints of the producers, workers and consumers that compose the economic system.

[17]There is evidence from Sraffa's diaries that they met frequently while Hicks was in Cambridge (from 1935 to 1938). There is also the draft of the letter from Sraffa to Hicks in which Sraffa makes a precise quote in relation to Hicks' "The Theory of Wages", first Edition, Hicks (1932). In a personal conversation, Massimo Di Matteo at the University of Siena, who has been a student of Hicks, told the first author that he had indeed asked Sraffa about what the 'useful criticisms' were. He recalls Hicks' answer that they discussed matters related with Walrasian General Equilibrium and Pareto.

economic system, which solely determine distribution at full employment. For Sraffa, General Equilibrium prices are not determined by the mechanical forces of the markets alone.[18]

4 Towards a Sraffian Monetary Theory of Production and Distribution

We have argued in the previous sections that monetary issues are an important subject of Sraffa's research agenda and that it could explain some of the theoretical positions that were adopted in his later contributions. We also outlined the explicit and implict role that money plays in PCMC. In order to build a monetary theory of production and distribution in the Sraffian vein, we first need to present the motivations for introducing money inside the Sraffian schemes. Further, we need to clarify what we mean by 'money' in this context and outline how the structure presented in PCMC can be suitably modified for this purpose.

It is well known that PCMC, as the subtitle indicates, was a *Prelude to a Critique of Economic Theory*. This critique is based on the idea of a self-replacing system and the associated notions of self-replacing prices, wages and profit rates. Self-replacing is the situation where production can continue to take place as it did during the previous production cycle. This condition is important because it allows one to focus on prices and distribution of the surplus in a situation where produced physical quantities are not changing. Sraffa has shown that the *natural prices* of the commodities in such a situation are not unique and they simultaneously determine the distribution of the physical surplus. Additionally, the values of gross or net output and the means of production (the value of capital) are themselves a function of prices and hence of distribution.

There are several directions in which this framework can be and have been extended in the literature, ranging from the introduction demand in to this framework, exploring compatibility with the Keynesian system, to devising a complete general equilibrium system. In the next subsection, we outline the motivations for generalising this framework to accomodate a form of money.

4.1 Money in the Sraffain Schemes: some motivations

Right from the outset, we would like to acknowledge that there have been attempts in the past to introduce money inside the Sraffian schemes (Panico, 1988a; Pivetti, 1991;

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[18]See also Panico (1988b, p.26) in support of this interpretation:

> A well-defined thread, going throughout his writings up to Production of Commodities by Means of Commodities, can thus be found in Sraffa's work. His theoretical interests were enhanced by the need to provide firm analytical foundations for his original view of the working of the economic system. This view emphasised the role of state intervention in determining income distribution and the equilibrium position of the economy, and the fact that policy decisions are influenced by the pressures of the dominant groups, and are not the result of purely technical considerations. His earlier analyses on the influence of monetary policies on income distribution were closer to the classical and the Marxist tradition than to the neoclassical one. The study of these analyses, together with that of his contacts with Keynes, thus provides useful insights in the evolution of Sraffa's thought, underlining that his later work is not 'merely abstract exercise in pure theory'.

Ciccarone, 1998). However, to the best of our knowledge, the possibility of generalising this framework by the introduction of money is far from being realised. There are several motivations behind the need for introducing money inside the Sraffian schemes.

First, money is an essential aspect of the capitalistic mode of production. This point hardly seems to need any elaboration and this case is eloquently made by Sraffa himself in his review of Hayek's book. Second, the fact that money may not have been necessary for Sraffa's critique of marginalist theory, which in fact was the main thrust of PCMC, does not automatically imply that there is no place for money in his entire theoretical apparatus. It seems useful to extend this scheme - not necessarily to get a complete theory - in order to see its potential, yet remaining faithful to the spirit in which Sraffa constructs his theoretical system. Third, if we view money not as a commodity like other commodities in the economic system, then the role it plays in the facilitating production needs to be studied. Fourth, there is both a need and scope for money - viewed as credit or a deferred means of payment - within Sraffian schemes.

Prices in Walrasian general equilibrium are those that are associated with equilibrium exchanges: no trade is allowed outside equilibrium.[19] Within Sraffian schemes the endowments are quantities in the possession of agents after the harvest and before the beginning of the market day. Evidently, the self-replacing prices are market clearing prices. Here the production (and consumption) plans are implemented due to the fact that producers and workers have the necessary purchasing power to buy the necessary means of production and the surplus, i.e. the commodities not used in production. That is, if deferred means of payments do not exist or are not generated, during the market day the prices must be such that the revenues of producers and workers must exactly match their expenditures (i.e., the excess supply and excess demand functions are zero).

The necessity of money for the exchanges is well explained by Arrow and Hahn (1971):

> The terms in which contracts are made matter. In particular, if money is the good in terms of which contracts are made, then the prices of goods in terms of money are of special significance. This is not the case if we consider an economy without the past and the future.[. . .] **If a serious monetary theory comes to be written, the fact that contracts are indeed made in terms of money will be of considerable importance** (Arrow and Hahn, 1971, pp.365-7, emphasis added).

As prelude to a critique, the assumption of uniform rate of profits and the absence of monetary means of exchange (which imply accounting equilibrium prices) are more than justified. The relevance of Sraffa's results have been questioned by Hahn (1982), among others, on the ground that the uniform rate of profits assumption present in PCMC reduces Sraffian schemes to be just a special case of the Walrasian General Equilibrium. In a recent paper (Zambelli, 2018b), the assumption of the uniform rate of profits has been removed and the properties of the Sraffian system have been studied under more general conditions (See Appendix A).

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[19]After twelve chapters, defining and proving theorems about Walrasian general equilibrium, Arrow and Hahn (1971) state this clearly:

> Of course, our model is in no shape to give a satisfactory formal account of the role of money. In particular it would be hard to "explain" the holding of money or why it mediates in most acts of exchange (p. 338).

Building on this, here we argue for an extension where there is a possibility of having exchanges of real goods or services against deferred means of payments (money, credit and debt). In order to introduce money in the Sraffian scheme, we can consider cases in which prices alone are not enough to allow for self-replacing. There is a distinct possibility that commodities may be exchanged due to the issue of new loans (credit and debt) or through the transfers of fiat money. The introduction of deferred means of payments inside Sraffian schemes provides a good starting point for introducing money (See Appendix B for a brief sketch of how this can be accomplished).

4.2   Marx, M-C-M and the capitalistic mode of production.

Although Sraffa's work has been widely interpreted as a critique of marginalism, it is also seen by some as a critique of the embodied labour theory of value. In Chapter VI, *Reduction to Dated Quantities of Labour* Sraffa is very clear about this. Sraffa demonstrates that the embodied labour terms[20] when summed up cannot be used as an invariable measure of capital and this has been seen as a critique of Ricardo's labour theory of value and its use by Marx. Most importantly, he shows that it cannot be reconciled with 'period of production' as an independent measure of the quantity of capital.[21] Sraffian schemes are sometimes viewed not just as a critique but also as a solution of Ricardo's embodied labour value problem and as a way to address the Marxian transformation problem (Marx, 1894, Capital, volume III, Ch. XII).[22]

Marx characterizes economic systems with two circular processes: the Commodity-Money-Commodity, $C–M–C$, and the Money-Commodity-Money, $M–C–M$.

> In the circuit $C–M–C$, the money is in the end converted into a commodity, that serves as a use-value; it is spent once for all. In the inverted form, $M–C–M$, on the contrary, the buyer lays out money in order that, as a seller, he may recover money. ...He lets the money go, but only with the sly intention of getting it back again. The money, therefore, is not spent, it is merely advanced (Marx, 1867, vol. 1, p.148)

There are those that interpret Sraffa's contribution as holding an *'antagonistic'* view with respect to Marx's work (Pilling, 1972; Lebowitz, 1973; de Brunhoff, 1975, 1990; Rowthorn, 1974; Nicholas, 2014) and others who see it as a *'harmonious'* one (Dobb, 1975; Eatwell, 1974; Laibman, 1975; Garegnani, 1978; Lippi, 1979; Hodgson, 1982; Steedman, 1977).

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[20]The quotation we refer to in sec. 3.2 is made at the end of section 48 in PCMC. Fig.2 in page 36 of Sraffa's book shows the value of different 'labour terms' which when summed up make the value in terms of 'embodied labour' of capital, which varies as the profit rate (i.e. distribution of the surplus) varies. This result is also stressed with the aid of Fig. 3, which is found in the same page as the above quotation.

[21]There is a possibility here for a misleading conclusion that this critique of economic theory that is based on the impossibility of measuring capital, or the values in general, is restricted to the adherence of a labour theory of value. This would be incorrect. Zambelli (2018b), for example, has shown that the values, even when the prices and values are determined by taking the surplus as the *numéraire*, are a function of distribution. Furthermore, when the agricultural sector (or any other sector) is taken to be the *numéraire*, there is no simple, unique measure of capital that is independent of distribution. This is a 'modern' critique and does not depend on the labour theory of value and the Austrian notion of the 'period of production' (Zambelli, 2018b) .

[22]For a contemporary discussion of the transformation problem see also, among others, Dobb (1967), Laibman (1973), Samuelson (1971), Seton (1957), Shaikh (1984), Steedman (1977), Winternitz (1948).

The *'antagonists'* do recognize that Marx had some *unsolved* issues in his analysis (i.e., the relation between commodity, money and capital when defined as social necessities)[23]. However, they consider the Sraffian solution to the transformation problem (and value theory) strictly as a solution to Ricardo's search for an *'invariable measure of value'*, but not as a solution to Marx's value problem.

The major reason for this would be that commodity prices, money and capital can perhaps be studied in the form in which they may be observed from a pure economic theory point of view (as in Ricardo). However, when discussing value, according to Marx, one has to consider the inescapable role of money in commodity circulation. In the capitalistic mode of production, the change from the *use value* of the commodities into their *exchange value* means a change in focus from the *C-M-C* circuit to the capitalistic circuit *M-C-M*. Hence Sraffian schemes as they stand may shed light on the *C-M-C* circuit, but cannot yet shed light on the *M–C–M* circuit. However, according to Marx, the latter characterizes the capitalistic mode of production, because *it is money which is the form that the commodities take.*[24] A formal treatment of money in the context of Sraffian schemes may be necessary if one aims at shedding light on the mechanics of the economic aspects of both C-M-C and M-C-M circuits and of the Marxian value theory in general.

We conclude with the suggestion by Nuti (1971, p.33) reported in the introduction of Panico (1988a, p.7):

> The most appropriate way of approaching the theory of distribution, reintroducing the reality of class struggle into this important branch of Political Economy, seems therefore that of combining the Sraffian relation between wage and profit rates with the little we know - not least from Marx - about the interaction of real and monetary phenomena.

We undertake first steps in this direction in our companion piece (Venkatachalam and

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[23]Pilling (1972); de Brunhoff (1973, 1975) discuss the difference between Ricardo (and a Ricardian interpretation of the work of Sraffa made by the neo-Ricardians) and Marxian theory of value as it is dealt with in Marx's Capital. *"For Marx did not attempt to construct a pure economic theory, his field of problems and his point of departure is quite different from that of Ricardo and that of pure economics"* (de Brunhoff, 1973, p.423). Crucial points are i) Marx's distinction between 'labour-time', 'abstract labour', 'labour force power' and 'socially determined labour' and ii) the different, unique meaning given to money.

[24]Marx's critique to Ricardo on this point may be summarized by the following:

> With him [Ricardo], however, wage labour and capital are again conceived as a natural, not as a historically specific social form [*Gesellschaftsform*] for the creation of wealth as use value; i.e. their form as such, precisely because it is natural, is *irrelevant*, and is not conceived in its *specific* relation to the form of wealth, just as wealth itself, in its exchange-value form, appears as a merely formal mediation of its material composition; thus the specific character of bourgeois wealth is not grasped precisely because it appears there as the adequate form of wealth as such, and thus, although *exchange value* is the point of departure, the specific economic forms of exchange themselves play no role at all in his economics. Instead, he always speaks about distribution of the general product of labour and of the soil among the three classes, as if the form of wealth based on exchange value were concerned only with use value, and as if exchange value were merely a ceremonial form, which vanishes in Ricardo just as money as medium of circulation vanishes in exchange. Therefore, in order to bring out the true laws of economics, he likes to refer to this relation of money as a merely formal one. This does not mean that commodities do not have *use values, but it is money which is the form that the commodities take.* Hence also his weakness in the doctrine of money proper (de Brunhoff, 1973, p.429, author translation from Marx's *Grundrisse*).

Zambelli, 2020) and a concise summary is presented in Appendix B. More importantly, we hope to have convinced the reader that this is a journey that seems worth undertaking and that it promises to offer interesting insights.

A   Appendix. Sraffian Schemes and Distribution

A.1   Production and methods of production

In PCMC (p.3, §1 and p.10, §9) Sraffa assumes that there is *'an annual cycle of production with an annual market'*.

At the end of the production cycle there are n produced commodities:

$$\mathbf{b} = [b_1, b_2, \ldots b_i, \ldots, b_n]^T \tag{1}$$

$i = 1, 2, \ldots, n$.

The method of production (PMCM, p.3, §1 and p. 6 §4) producing b_i is a linear combination of means of production and labour:

$$a_i^1, a_i^2, \ldots, a_i^j, \ldots, a_i^n, \ell_i \to b_i \tag{2}$$

where a_i^j denotes the means of production produced by industry j used in the production of commodity i and ℓ_i the labour used in the production of b_i.

In a more compact form, the method of production i may be written as $\mathbf{a_i}, \ell_i \to b_i$.

A.2   Surplus to be distributed

The *'economy produces more than the minimum necessary for replacement and there is a surplus to be distributed'* (PCMC, p.6, §4).

That is,

$$
\begin{aligned}
s_1 &= b_1 &- &\quad \sum_{i=1}^{n} a_i^1 \\
s_2 &= b_2 &- &\quad \sum_{i=1}^{n} a_i^2 \\
\vdots &= \vdots &- &\quad \vdots \\
s_j &= b_j &- &\quad \sum_{i=1}^{n} a_i^j \\
\vdots &= \vdots &- &\quad \vdots \\
s_n &= b_n &- &\quad \sum_{i=1}^{n} a_i^n
\end{aligned}
\tag{3}
$$

where s_i is the surplus of commodity i available for distribution after the quantities $\{a_i^j\}$ have been put aside for the next year's production. Alternatively, it is the quantity produced in the previous period which is left over, once the inputs used in production have been removed.

In compact matrix notation, we have

$$\mathbf{S} = (\mathbf{B} - \mathbf{A})^T \mathbf{e} \tag{4}$$

where: $\mathbf{e}$ is the $n \times 1$ unit or summation vector (each element is 1); T is the transpose operator; $\mathbf{S}$ is the $n \times 1$ Physical Surplus vector or Physical *Net National Product*; $\mathbf{B}$ is the $n \times n$ diagonal matrix composed of gross production $\mathbf{b}$ as its diagonal elements.

A.3   Self-replacing prices

Sraffa also assumes, for most of his book, that wage is paid *post factum* (PCMC, 9-10, §8-9). Given the knowledge of the methods of production used during the previous annual production cycle (and assuming that the same methods will be used), Sraffa searches

for uniform prices that would allow the system to replicate these commodities during the next production cycles. These prices, following the classical tradition, could have different names (PCMC, p. 7-8, §8). Here we refer to them as self-replacing prices.

If this is our problem setup, the accounting balance would require that:

$$
\begin{aligned}
(1 + r_1)\mathbf{a}_1\mathbf{p} + \ell_1 w &= b_1 p_1 \\
(1 + r_2)\mathbf{a}_2\mathbf{p} + \ell_2 w &= b_2 p_2 \\
\dots &= \dots \\
(1 + r_i)\mathbf{a}_i\mathbf{p} + \ell_i w &= b_i p_i \\
\dots &= \dots \\
(1 + r_n)\mathbf{a}_n\mathbf{p} + \ell_n w &= b_n p_n
\end{aligned}
\tag{5}
$$

where: $\mathbf{p} = [p_1, p_2, \dots, p_n]^T$ is the price vector, $r_1, r_2, \dots, r_n$ are the sectoral profit rates and w is the the uniform wage rate.

In matrix notation, eqs. 5 become

$$
(\mathbf{I} + \mathbf{R})\mathbf{A}\mathbf{p} + \mathbf{L}w = \mathbf{B}\mathbf{p}
\tag{6}
$$

where: $\mathbf{I}$ is the identity $n \times n$ matrix; $\mathbf{R} = diag(\mathbf{r})$ is the diagonal matrix, whose diagonal elements are the rate of profits in each single industry, $r_1, r_2, \dots, r_i, \dots r_n$ (vector $\mathbf{r}$); $\mathbf{A}$ is the $n \times n$ matrix denotes the means of production $\{a_i^j\}$; $\mathbf{L}$ is the $n \times 1$ vector whose elements $\{\ell_i\}$ are the labour used in production.

A.4 Number of equations and number of variables

The system of equations 5 (or 6) is indeterminate. There are at least two cases that we can consider.

i) **Non-uniform rates of profits:** in this case, there are $2 \times n + 1$ variables (n prices, n profit rates and the wage rate w) and n equations. This is a more general case and it is examined in Zambelli (2018b).

ii) **Uniform rate of profits:** this is case that Sraffa examined in PCMC. Sraffa simplifies the problem by assuming that the rates of profits are uniform. In that case, the n rates of profits are asummed to be equal to a single rate of profits: $r = r_1 = r_2 = \dots = r_n$ (PMCM, p.11, §11)[25]. Here there are n equations and the number of variables are reduced to $n + 2$ (i.e., n prices, a wage rate w and a uniform rate of profits r). Hence the system is still indeterminate.

A.5 Net national income in a self-replacing system

A.5.1 The share between workers and producers of the surplus in value terms

'The national income of a system in a self-replacing state consists of the set of commodities which are left over when from the gross national product we have removed item by item the articles which go to replace the means of production used up in all industries [the surplus vector $\mathbf{S}$]. The value of this set of commodities [...] we make equal to unity' (PCMC, p.11, §12).

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[25]Zambelli (2018b) discusses this assumption. The uniform rate of profits is obviously a special case of eqs. 5. This assumption is justified in PCMC, which is meant as a *Prelude to a Critique of Economic Theory*.

Formally, the value of the Net National Product (Y^{NNP}) is:

$$Y^{NNP} = \mathbf{S}^T \mathbf{p} = 1 \tag{7}$$

If the value of the Net National Product is unity, this implies that $\mathbf{S}^T\mathbf{p} = 1$ is the *numéraire*.

In value terms, the value of the surplus that goes to producers is $\sum_{i=1}^{n}(r_i\mathbf{a}_i\mathbf{p}) = \mathbf{e}_{n\times1}^T\mathbf{RAp}$. Similarly, the value of the surplus that goes to the workers is $\sum_{i=1}^{n}(w\ell_i) = \mathbf{e}_{n\times1}^T w\mathbf{L}$:[26]

$$Y^{NNP} = \overbrace{\mathbf{e}_{n\times1}^T\mathbf{RAp}}^{\substack{\text{Share to}\\\text{Producers (value)}}} + \overbrace{\mathbf{e}_{n\times1}^T w\mathbf{L}}^{\substack{\text{Share to}\\\text{Workers (value)}}} \tag{8}$$

For the special case in which the rate of profits is uniform, as in PCMC, we have:

$$Y^{NNP} = \overbrace{\mathbf{e}_{n\times1}^T r\mathbf{Ap}}^{\substack{\text{Share to}\\\text{Producers (value)}}} + \overbrace{\mathbf{e}_{n\times1}^T w\mathbf{L}}^{\substack{\text{Share to}\\\text{Workers (value)}}} \tag{9}$$

A.5.2 Distribution of the surplus to industries and workers: value and physical terms

The value of the Net National Product, $\mathbf{S}^T\mathbf{p}$ is distributed to the n industries and workers:

- $d_1\mathbf{S}^T\mathbf{p}$ is the share going to industry 1, $d_2\mathbf{S}^T\mathbf{p}$ is the share to industry 2, ..., $d_n\mathbf{S}^t\mathbf{p}$ is the share to industry n

- $d_w\mathbf{S}^T\mathbf{p}$ is the share that goes to the workers

Clearly $\sum_i^n d_i + d_w = 1$.

$$\mathbf{dS}^T\mathbf{p} = \overbrace{\begin{bmatrix} d_1\mathbf{S}^T\mathbf{p} \\ d_2\mathbf{S}^T\mathbf{p} \\ \vdots \\ d_n\mathbf{S}^T\mathbf{p} \\ d_w\mathbf{S}^T\mathbf{p} \end{bmatrix}}^{\substack{\text{Distribution of}\\\text{Surplus or NNP}}} = \overbrace{\begin{bmatrix} b_1p_1 - \mathbf{a}_1\mathbf{p} - \ell_1 w \\ b_2p_2 - \mathbf{a}_2\mathbf{p} - \ell_2 w \\ \vdots \\ b_np_n - \mathbf{a}_n\mathbf{p} - \ell_n w \\ \mathbf{e}^T\mathbf{L}w \end{bmatrix}}^{\substack{\text{Purchasing Capacity}\\\text{Income}}} = \overbrace{\begin{bmatrix} r_1\mathbf{a}_1\mathbf{p} \\ r_2\mathbf{a}_2\mathbf{p} \\ \vdots \\ r_n\mathbf{a}_n\mathbf{p} \\ \mathbf{e}^T\mathbf{L}w \end{bmatrix}}^{\substack{\text{Purchasing Capacity}\\\text{Income}}} \tag{10}$$

Given that $\mathbf{S}^T\mathbf{p} = 1$, the vector $\mathbf{d} = [d_1, d_2, \ldots, d_n, d_w]^T$ denotes distribution of the surplus $\mathbf{S}$ both in value and physical terms.

Recall that $\mathbf{S}^T\mathbf{p}=1$ (see eq. 7), the above accounting identity, eq. 10, may be written in a compact form as:

$$\mathbf{d} = \begin{bmatrix} \mathbf{d}_{n\times1} \\ d_w \end{bmatrix} = \begin{bmatrix} (\mathbf{B} - \mathbf{A}) & -\mathbf{L} \\ \mathbf{0}_{1\times n} & \mathbf{e}^T\mathbf{L} \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ w \end{bmatrix} = \begin{bmatrix} \mathbf{RA} & \mathbf{0}_{n\times1} \\ \mathbf{0}_{1\times n} & \mathbf{e}^T\mathbf{L} \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ w \end{bmatrix} \tag{11}$$

---

[26]$\mathbf{e}_{n\times1}$ is the summation vector

Clearly, when the self-replacing prices $\mathbf{p}$ and wage rate w are given, the distribution is uniquely determined and the profit rates $\mathbf{r}$ are determined as well. Also, when the distribution $\mathbf{d}$ is given, prices $\mathbf{p}$ and the wage rate w are also determined. In this case we have that:

$$\left[\begin{array}{c} \mathbf{p} \\ w \end{array}\right] = \left[\begin{array}{cc} (\mathbf{B} - \mathbf{A}) & -\mathbf{L} \\ \mathbf{0}_{1 \times n} & \mathbf{e}^T \mathbf{L} \end{array}\right]^{-1} \mathbf{d} = \left[\begin{array}{cc} (\mathbf{RA})^{-1} & \mathbf{0}_{n \times 1} \\ \mathbf{0}_{1 \times n} & (\mathbf{e}^T \mathbf{L})^{-1} \end{array}\right] \mathbf{d} \qquad (12)$$

From eq. 11 and eq. 12, we see that distribution and prices are determined **simultaneously**.

The vector $\mathbf{d}$ is the physical distribution of the surplus $\mathbf{S}$ to producers and workers, but it is also the distribution in value terms, i.e., the share in value of the net national product shown in eq. 8. The share going to the producers is given by $\mathbf{e}^T \mathbf{RAp}$ and that which goes to the workers is given by $\mathbf{e}^T \mathbf{L}w$.

Furthermore, Zambelli (2018b, Appendix A) shows an equivalence between the actual physical distribution and the equivalent distribution in terms of a proportion of the surplus vector $\mathbf{S}$. In other words, if $d_i > d_j$, it implies that $d_i \mathbf{S}^T \mathbf{p} > d_j \mathbf{S}^T \mathbf{p}$ and it as if $d_i \mathbf{S} > d_j \mathbf{S}$. This allows for a physical comparison of the distribution of the surplus.

A.6 Indeterminacy or exogeneity

In both cases i) and ii) mentioned earlier, the system can be closed only by adding some additional exogenous elements or additional theories. Here we have chosen the physical surplus $\mathbf{S}$ as the *numéraire*. The economic system is described in terms of n equations (eqs. 5) and the *numéraire* equation (eq. 7).

This implies that when considering a more general case with non-uniform rates of profits, we have $n + 1$ equations and $2n + 1$ variables. Therefore, there are n degrees of freedom: i.e., all the self-replacing prices, rates of profits and wage rate cannot be computed by the knowledge of the quantities and the methods of production alone. In order to solve the system of equations, n variables among $\{p_1, p_2, \ldots, p_n, r_1, r_2, \ldots, r_n, w\}$ have to be given exogeneously, i.e from outside the economic system.

When we consider Sraffa's special case with uniform rate of profits (case ii), we have $n + 1$ equations (i.e., eqs. 5 is restricted to the case in which the rates of profits are uniform and the *numéraire* eq. 7) and $n + 2$ variables $\{p_1, p_2, \ldots, p_n, r, w\}$. In this case, there is only one degree of freedom and one of the $n + 2$ variables have to be exogenously given.

B Incorporating credit and debt inside a Sraffian framework: an outline

We now outline the way in which we can introduce money, credit and debt, inside the Sraffian schemes (For details, see our companion paper Venkatachalam and Zambelli (2020)).

When we discuss an economic system, whether real or virtual, we often deal with a world of bilateral or multilateral exchanges. In such a world, when someone buys a good, there is always a counterpart who sells. In the traditional Sraffian schemes, the possibility of deferred means of payment have not been adequately explored. For the aims and arguments developed in PCMC, the introduction of deferred means of payments are not strictly necessary and their omission may be understandable. But as claim in sec. 3, money and deferred means of payments are indeed important and that there is a clear scope to extend the framework presented in PCMC by incorporating money, credit and debt.

In appendix A, we provided a formal treatment of generalized Sraffian schemes, where the rates of profits are non-uniform (system presented in PCMC with uniform rate of profits is a special case). As we have discussed above, when the system is indeterminate, there is ample space in theory for including elements that are not typically considered as belonging to the realm of economics.

Consider the more general case where the rates of profits are allowed to be non-uniform. We have $n + 1$ equations and $(2n + 1)$ the variables (see appendix A.6). If we now assume that the commodity prices $\bar{\mathbf{p}}$ and wage rate $\bar{w}$ are exogenously given we have the following inequalities:

$$
\begin{array}{ccc}
\overbrace{\begin{array}{l}
a_1^1 \bar{p}_1 + \ldots + a_1^j \bar{p}_j + \ldots + a_1^n \bar{p}_n + \ell_1 \bar{w} \\
a_2^1 \bar{p}_1 + \ldots + a_2^j \bar{p}_j + \ldots + a_2^n \bar{p}_n + \ell_2 \bar{w} \\
\vdots + \vdots + \vdots + \vdots + \vdots + \vdots \\
a_i^1 \bar{p}_1 + \ldots + a_i^j \bar{p}_j + \ldots + a_i^n \bar{p}_n + \ell_i \bar{w} \\
\vdots + \vdots + \vdots + \vdots + \vdots + \vdots \\
a_n^1 \bar{p}_1 + \ldots + a_1^j \bar{p}_j + \ldots + a_n^n \bar{p}_n + \ell_n \bar{w}
\end{array}}^{\substack{\text{Virtual} \\ \text{Expenditures}}}
& \begin{array}{c} \lessgtr \\ \lessgtr \\ \\ \lessgtr \\ \\ \lessgtr \end{array}
& \begin{array}{c}
\overbrace{b_1 \bar{p}_1}^{\substack{\text{Virtual} \\ \text{Revenues}}} \\
b_2 \bar{p}_2 \\
\vdots \\
b_i \bar{p}_i \\
\vdots \\
b_n \bar{p}_n
\end{array}
\end{array}
\tag{13}
$$

In matrix notation, this could be written as:

$$
\overbrace{\mathbf{A}\bar{\mathbf{p}} + \mathbf{L}\bar{w}}^{\substack{\text{Virtual} \\ \text{Expenditures}}} \quad \lessgtr \quad \overbrace{\mathbf{B}\bar{\mathbf{p}}}^{\substack{\text{Virtual} \\ \text{Revenues}}}
\tag{14}
$$

The prices and the wage rate in the above equations are to be seen as virtual or bookkeeping prices, which may or may not be the actual exchange prices. In fact, they cannot be the actual exchange prices when there is at least one industry for whom the bookkeeping expenditures (left-hand side) would be greater than bookkeeping revenues (right-hand side). In this case, the rate of profits of this industry is obviously negative.

This is a situation where there would not be enough purchasing power for the exchanges to take place and some industries would be left with unsold commodities. This, in turn,

would mean that the economic system as a whole would not be in the self-replacing condition.

Clearly, the industries with virtual expenditures higher than their virtual revenues would not have the necessary purchasing power to buy the means required to replicate production of the previous period. These industries, therefore, are in a condition where there is a potential financial deficit. Concurrently, there would be industries which would not be able to sell all of their product. The industries in 'financial deficit' would be able to purchase the necessary means of production only by agreeing to a deferred payment to take place during the years to follow. At the same time, the industries in potential 'financial surplus' would be able to sell all of their product by agreeing to deferred payments by the borrowers. The traditional Sraffian schemes outlined in Appendix A can be extended or generalised to accomodate this situation.

We can conveniently view these deferred means of payments as a form of an I Owe You (IOUs). This can be seen to include all forms of financial contracts, where there is a promise to return goods in the future. These obligations may be with an explicit delivery date (as in the case of forward contracts associated with real goods), or with a relatively loose delivery date (as in the case of standard means of exchange such as cash, checks, debt and credit accounts, bonds etc.).

We can consider a situation wherein a commodity is exchanged for a promise to pay back at a future point in time (deferred payment). The entity selling this commodity would see its credit increasing and correspondingly, the counterpart buying this commodity would have its debt increase. This might take place by the writing off of means of payments previously generated or by issuing new means of payments. Furthermore, this could take place through an institution, like the banking system, or through direct contracts. A full-fledged treatment of Sraffian schemes with deferred means of payments can be found in our companion piece Venkatachalam and Zambelli (2020).

Given this structure, we can ask several interesting questions and perform thought experiments. To start with, we can explore the set of prices and possible financial conditions that would allow the system to reproduce itself, if one allows for transfer of purchasing power in the form of credit and debt. In other words, identifying the set of prices that would allow self-replacing. Note that IOUs may be generated and transferred from one period to another, which in turn might influence the set of self-replacing prices in the subsequent periods. These may offer us insights concerning the potential paths that are available for the system to evolve in the future. To answer questions of this nature, it is also important to study the effects that already existing future promises to pay may have on determining the set of self-replacing prices. All of this require keeping a close track of the credit and debt structure with careful accounting built in to the framework.

We also note that in our attempts to introduce money, we follow Sraffas approach in PCMC and do not provide a theory of prices and distribution. Instead, the objective would be to determine, for given past methods of production, usage of labour and output, those set of prices and the possible financial conditions that would allow the system to reproduce itself. We hope that this provides some pointers to the direction in which Sraffian schemes can be generalised.

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