Productivity accounting in vertically (hyper-)integrated terms: bridging the gap between theory and empirics☆

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Abstract

The present paper is a methodological contribution introducing a disaggregated physical productivity accounting framework in vertically (hyper-)integrated terms, establishing a direct correspondence between Supply-Use Tables and Pasinetti's (1973; 1988) theoretical magnitudes. As an empirical application, we computed productivity indicators and indexes of direction of technical change at the subsystem level for the case of Italy during 1999-2007. Our findings suggest that: (a) only 60% of productivity growth accrued to real wages; (b) the degree of mechanisation increased; (c) the most dynamic subsystems correspond to consolidated sectors; and (d) technical change has almost always been capital intensity increasing.

Keywords: Total Labour Productivity, Vertical (hyper-)integration, Input-Output Analysis

JEL Classification: C67, B51, D24, O14, O41

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1. Introduction

Productivity analysis within the Classical tradition has frequently departed from the insight that partial industry measures cannot adequately deal with the interdependent character of technical change, while system measures involving total (direct and indirect) input requirements per unit of *net* output do fulfil this essential pre-requisite (see, e.g. Gupta and Steedman, 1971). The idea that observing only direct labour-saving trends could be misleading has been early recognised (Leontief, 1953, pp. 38-40) and explicit attempts to recursively measure input requirements from every supporting industry to produce each component of the net product have been conceived at an early stage as well (see, e.g. Vincent, 1962).[3]

But only the explicit notion of a net output subsystem (Sraffa, 1960, Appendix A) provided a precise alternative description of the technique in use, which allowed for a disaggregated analysis of technical change in physical terms (see, e.g. Pasinetti, 1963; Gossling, 1972). The conceptual apportionment of gross outputs, means of production and quantities of labour into (relatively) autonomous parts — each reproducing system-wide interdependence due to circularity — allowed to overcome the problem of aggregation, which usually characterises sectoral measures of productivity changes.[4]

In this sense, the compact representation of self-replacing subsystems in terms of vertically integrated sectors (Pasinetti, 1973) can operationalise the period-by-period mapping between industries and subsystems, shifting the disaggregated unit of analysis for the purpose of quantifying the over-all *effects* of technical change.[5]

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[3]For example, in discussing the problem of the influence of compositional changes in final demand on the measurement of technical change, Leontief thought that the solution would be that of "computing for each one of the different structural situations a complete set of A's [input coefficients], each showing the dependence of one particular output on *one individual component* of the final demand" (Leontief, 1953, p. 40, italics added).

[4]See, for example, Steedman (1983).

[5]In fact, it should always be kept in mind that "the analytical device of vertical integration *is not meant* to catch the detailed and localized sources of technical change; on the contrary, it is meant to synthesize the overall *effect* of technical change" (Pasinetti, 1990, p. 258).

The use of vertically integrated sectors within productivity studies has been widespread (see, e.g. Rampa, 1981; Ochoa, 1986; Seyfried, 1988; Elmslie and Milberg, 1996; De Juan and Febrero, 2000). To our knowledge, however, there are, at least, four key issues which have not been dealt with in the literature:

(1) the explicit recognition that observed empirical structures contain a growth/decay element, so that self-replacement and expansion requirements in Pasinetti (1973) have to be accurately specified,[6]

(2) the need to conceptually and empirically distinguish between depreciation (an income side magnitude) and replacement needs (a physical notion) in the construction of subsystems including fixed capital inputs,[7]

(3) the need to provide an empirical counterpart to growing subsystems or vertically hyper-integrated sectors, as formulated by Pasinetti (1988), and

(4) the necessity to assess the consequences of shifting from a single product system towards a pure joint products framework, compatible with a set of Supply-Use tables.[8]

As regards the first point, data limitations may render impossible to perform an empirical separation between what re-enters the circular flow to replace productive capacity and what contributes to the expansion/contraction of the economy, whereas an explicit formulation in terms of measurable empirical magnitudes is required. As to the second issue, a distinction between depreciation and physical replacements is necessary, given that productivity accounting relies on the expenditure side of the system, and not on the income (or value added) side. Thirdly, the empirical formulation of growing subsystems can operationalise the device of vertical *hyper*-integration, which is of particular interest in a dynamic setting.

[6]Exceptions can be found in Lager (1997, 2000), though these contributions do not deal with productivity accounting.

[7]Instead of concentrating on physical replacements, applied studies usually focus on depreciation, even to assess *physical* productivity changes. See, e.g. Gupta and Steedman (1971); Ochoa (1986); De Juan and Febrero (2000); Flaschel et al. (2013).

[8]See Soklis (2011) in this direction, though concerning the empirical computation of wage-profit curves, and involving circulating capital only.

Finally, the shift towards a Supply-Use framework needs to be considered in some more detail. Usually, contributions in this field start from a single-product *model*, derive theoretical indicators, and eventually adapt empirical data to fit (with varying accuracy) the theoretical concepts. This paper proceeds differently. We depart from an *empirical* Supply-Use scheme, corresponding to a square commodity × industry system, and with this framework we directly construct subsystems and productivity indicators, gradually establishing a mapping *from* empirical magnitudes *to* theoretical concepts. In this way, all complexities involving joint products, valuation schemes, imported commodities and activity levels, among others, are dealt with immediately. Hence, for example, instead of adopting an Input-Output technology assumption to do away with joint production, we deal with the problems that joint products bring into the picture (e.g. the empirical non fulfilment of the all-productiveness property).

Essentially, the set of Supply-Use Tables of the System of National Accounts (SNA, hereinafter) allows to make a precise separation between (commodity) prices and volume changes, which is a crucial pre-requisite for setting-up an accurate disaggregated physical productivity accounting scheme.[9] But taking the SNA as an explicit point of departure intends to convey a further message. From a Classical standpoint, the evolution of the theoretical position of the SNA should raise worrying awareness. While the SNA-1968 (partially drafted by Richard Stone) contained a set of productivity indicators deeply connected to the Classical tradition (UN, 1968, pp. 66-70), the latest SNA-2008 has completely done away any formulation which does not comply with the logic of Neoclassical Multi-Factor Productivity (MFP) (UN, 2009, p. 412). In this light, formulating a Classical productivity accounting scheme using the empirical elements provided by the SNA-2008 seems to be utterly justified.

[9]This is not possible by adopting any of the main Input-Output technology assumptions, as is argued in detail below. For an early discussion of this issue as regards the 'industry' and 'commodity' technology assumptions, see Flaschel (1980). In fact, "[m]easures of sectoral and total labour productivity should be based on technological data as much as possible (subject to an unavoidable degree of aggregation) and they should not *definitionally* depend on price variables" (Flaschel et al., 2013, p. 380).

The aim of the present paper is therefore to set-up a framework for physical productivity accounting in terms of vertically integrated and hyper-integrated sectors, introducing disaggregated (subsystem-specific) measures of productivity changes and of the direction of technical change that do not depend on relative prices (and thus, income distribution) and on the composition of the net product. To illustrate the use of the proposed framework, we provide an application for the Italian economy during 1999-2007. It should be kept in mind, however, that the focus of this paper is primarily methodological.

After this introduction, we proceed by formulating vertically integrated and hyper-integrated sectors in terms of the empirical categories emerging from a set of Supply-Use Tables (Section 2). Measures of productivity changes and of the direction of technical change are introduced in Section 3. Section 4 reports and discusses the empirical results. Finally, Section 5 summarises and concludes.

2. Alternative descriptions of the technique in use

In order to give an analytical formulation of vertically integrated and hyper-integrated sectors in terms of the empirical categories of a set of Supply-Use Tables, we depart from commodity balances by source of demand for domestically produced items, i.e. the physical counterpart to the expenditure side of an Input-Output system. Gross outputs in physical terms are identically equal to:[10]

$$\mathbf{q} \equiv \mathbf{V}_q \mathbf{e} \equiv \mathbf{U}_q \mathbf{e} + \mathbf{F}_{k_q} \mathbf{e} + \mathbf{c} \tag{1}$$

[10] Appendix A specifies matrix notation for all those symbols which are not explicitly introduced in the main text. Notation has been chosen in order to keep the present formulation as close as possible to symbols frequently used in *empirical* Supply-Use schemes coming from the System of National Accounts.

where $\mathbf{c}$ contains all sources of expenditure that do not re-enter the circular flow of reproduction as productive capacity.[11]

2.1. Vertically integrated sectors

To establish a correspondence between commodity balances and the theoretical magnitudes appearing in Pasinetti's (1973) article on vertical integration we should distinguish self-replacement from expansion requirements, both for fixed and circulating capital inputs.

National accountants compute gross stocks of fixed capital by accumulating successive vintages of gross investment flows valued at a common price system. In each year, to every past flow corresponding to each type of capital good a survival probability is applied, in order to identify those durable instruments of production that have not been discarded. Hence, the difference between current and previous period stocks (valued at the same price system) will include both current gross fixed capital formation and retirements of capital items which (statistically) could not survive to this date.[12] Formally, for the current time period we have:

$$\mathbf{K}^*_q - \mathbf{K}^*_{q(-1)} = \mathbf{F}^*_{k_q} - \mathbf{R}^*_{k_q}$$

But given that $\mathbf{K}^*_q$, $\mathbf{K}^*_{q(-1)}$ and $\mathbf{F}^*_{k_q}$ are those magnitudes usually reported, fixed capital retirements are obtained as a residual:

$$\mathbf{R}^*_{k_q} = -(\mathbf{K}^*_q - \mathbf{K}^*_{q(-1)} - \mathbf{F}^*_{k_q})$$

Moreover, assume that durable capital inputs have a gestation period of one 'year'.[13]

[11] These sources include final private and government consumption as well as exports (even of intermediates or capital goods). Hence, in matrix notation: $\mathbf{f}_c = \widehat{\mathbf{p}}_s\mathbf{c} = \mathbf{f}_{c_p} + \mathbf{f}_g + \mathbf{f}_x$.

[12] See OECD (2009, pp. 38-40) for details.

[13] From a theoretical standpoint, this may seem an arbitrary assumption. However, note that the column of gross fixed capital formation in a typical Use Table includes *only* acquisition less disposal of fixed assets, while work-in-progress (which constitute production during the current accounting period in need of further processing to be saleable) are instead included in the vector of changes in inventories (for details, see

Hence, by subtracting retirements from gross fixed capital formation we are implicitly defining the concept of *new* investments, i.e. current expenditure on capital goods devoted to the expansion of productive capacity, which will be ready for operation in the following period. Formally, we have:

$$\mathbf{J}^*_{k_q} = \mathbf{F}^*_{k_q} - \mathbf{R}^*_{k_q} \tag{2}$$

The concept and meaning of new investments cast lights on two points. First, it allows to objectively think of retirements as replacement needs, i.e. an artificial apportionment of investment expenditure aimed at reconstituting productive capacity in place.[14] Second, it allows to make a crucial distinction with respect to the traditional concept of *net* investment, i.e. gross investment minus depreciation.

In fact, the reader might be puzzled as to why we have focused on replacement needs instead of depreciation to identify self-replacement requirements. Depreciation is a book-keeping concept, belonging to the value-added side of the economy. As such, it does not have a physical counterpart. According to book-keeping (linear) depreciation schemes, when a new machine is bought and enters productive capacity, in order not to alter the cost/profits relation in the corresponding accounting period, an estimate of its life-time is made. The value of the machine is then split into as many parts as its estimated life-time, and thus spread over the whole period, in order to smooth the associated increase in production costs. This has nothing to do with the purely physical concept of replacements, which instead pertains exclusively to the expenditure side of the system in physical terms.[15]

EUROSTAT, 2008, pp. 154-6). This means that current gross investments consist entirely of finished capital goods, which add to productive capacity.

[14]In principle, investment decisions clearly depend on a multiplicity of factors. However, given that our aim is to device an objective accounting framework, it is our contention that this should be done without recourse to behavioural hypotheses. As such, an apportionment of gross flows into a self-replacement and expansion component based on empirically given retirements seems to be an acceptable strategy.

[15]In his 1959 paper, Pasinetti had already noticed that "[a]ll quantities could be interpreted in *net* terms but, since depreciation allowances always contain elements of arbitrariness, it is better to work with gross quantities" (Pasinetti, 1959, p. 275). In fact, as regards physical productivity accounting, we couldn't agree more with Steindl reporting Leontief's view: "I remember a lecture by Leontief in which he said that

Additionally, national accounts generally do not render available gross stock and flow fixed capital matrices (of dimension "commodity of origin × industry of destination") separating domestic from imported sources, but only domestic and imported column vectors of gross investments by commodity ($\mathbf{f}_{k_q}$ and $\mathbf{f}^m_{k_q}$, respectively). Given that this separation is necessary for productivity accounting, we assumed, for all industries and for each fixed capital input, the same proportion of imported to domestic demand. Computationally:[16]

$$\mathbf{F}_{k_q} = \widehat{\boldsymbol{\theta}}_q \mathbf{F}^*_{k_q}, \quad \mathbf{R}_{k_q} = \widehat{\boldsymbol{\theta}}_q \mathbf{R}^*_{k_q}, \quad \widehat{\boldsymbol{\theta}}_q = \widehat{\mathbf{f}}_{k_q}(\widehat{\mathbf{f}}^*_{k_q})^{-1}, \quad \mathbf{f}^*_{k_q} = \mathbf{f}_{k_q} + \widehat{\boldsymbol{\epsilon}}_p \mathbf{f}^m_{k_q}, \quad \widehat{\boldsymbol{\epsilon}}_p = \widehat{\mathbf{p}}^m_s \widehat{\mathbf{p}}^{-1}_s$$

As regards circulating capital inputs, the analytical separation between self-replacement and expansion requires to compute, under the technique *currently* in use, those activity levels allowing each industry to reproduce gross outputs from the previous period. In matrix terms:[17]

$$\begin{aligned}
\mathbf{x}_{(-1)} &= \mathbf{V}_q^{-1}\mathbf{q}_{(-1)}, \quad \mathbf{q}_{(-1)} = \mathbf{V}_{q(-1)}\mathbf{e} \\
\mathbf{R}_{u_q} &= \mathbf{U}_q \widehat{\mathbf{x}}_{(-1)} \\
\mathbf{R}^*_{u_q} &= \mathbf{U}^*_q \widehat{\mathbf{x}}_{(-1)}
\end{aligned}$$

where $\mathbf{x}_{(-1)}$ is the activity level vector applied to current domestic and total Use matrices ($\mathbf{U}_q$ and $\mathbf{U}^*_q$, respectively) to obtain domestic and total circulating capital replacements ($\mathbf{R}_{u_q}$ and $\mathbf{R}^*_{u_q}$, respectively).[18] Thus, also in the case of circulating capital it is possible to define

depreciation is a concept used by the tax administration, it is not an economic concept at all" (Steindl, 1993, p. 121).

[16]Note that the vector of total fixed capital inputs by commodity ($\mathbf{f}^*_{k_q}$) depends on the diagonal matrix of terms of trade by commodity ($\widehat{\boldsymbol{\epsilon}}_p$). Hence, the estimated separation between domestically produced and imported gross investments ($\widehat{\boldsymbol{\theta}}_q$) depends on given base period terms of trade. See Appendix B for details on the relationship between physical and nominal magnitudes.

[17]Note that it is necessary to assume that the Make matrix $\mathbf{V}_q$ is non-singular.

[18]This procedure evinces a crucial difference between circulating and fixed capital. Whereas in the first case $\mathbf{R}_{u_q}$ is productively consumed during the current period — so the consequences of $t-1$ activity levels are

(total) expansion requirements as:

$$\mathbf{J}^*_{u_q} = \mathbf{U}^*_q - \mathbf{R}^*_{u_q} \tag{3}$$

At this point, introduce domestic fixed and circulating capital replacements into commodity balances (1):

$$\mathbf{V}_q\mathbf{e} = \mathbf{U}_q\mathbf{e} + \mathbf{F}_{k_q}\mathbf{e} + \mathbf{c} + (\mathbf{R}_{u_q}\mathbf{e} - \mathbf{R}_{u_q}\mathbf{e}) + (\mathbf{R}_{k_q}\mathbf{e} - \mathbf{R}_{k_q}\mathbf{e})$$

and re-order conveniently:

$$\mathbf{V}_q\mathbf{e} = (\mathbf{R}_{u_q} + \mathbf{R}_{k_q})\mathbf{e} + (\mathbf{U}_q - \mathbf{R}_{u_q})\mathbf{e} + (\mathbf{F}_{k_q} - \mathbf{R}_{k_q})\mathbf{e} + \mathbf{c}$$

Proceeding as in (2) and (3), domestic expansion requirements for fixed and circulating capital may be defined as $\mathbf{J}_{k_q} = \mathbf{F}_{k_q} - \mathbf{R}_{k_q}$ and $\mathbf{J}_{u_q} = \mathbf{U}_q - \mathbf{R}_{u_q}$, respectively, obtaining:

$$\mathbf{V}_q\mathbf{e} = (\mathbf{R}_{u_q} + \mathbf{R}_{k_q})\mathbf{e} + (\mathbf{J}_{u_q} + \mathbf{J}_{k_q})\mathbf{e} + \mathbf{c}$$

By further defining $\mathbf{R}_q \equiv \mathbf{R}_{u_q} + \mathbf{R}_{k_q}$ as the matrix of (domestic) replacement needs and $\mathbf{J}_q \equiv \mathbf{J}_{u_q} + \mathbf{J}_{k_q}$ as the (domestic) expansion matrix, we finally have:

$$(\mathbf{V}_q - \mathbf{R}_q)\mathbf{e} = \mathbf{y} \tag{4}$$

exhausted in t — this is not so with fixed capital. Investment consequences will probably carry on for many periods to come (which also means that current production conditions are influenced by lagged investment flows of several past periods). Hence, an artificial separation between self-replacement and expansion based *only* in activity levels of t and $t-1$ is out of place. Instead, we have proceeded by using empirical period-by-period gross stocks and flows of durable means of production. Alternatively, it could have been possible to discard all stock magnitudes and, for a given (final commodity-specific) steady growth path, reconstruct replacement and expansion requirements from flow magnitudes (see, for example, Lager, 1997).

where $\mathbf{y} = \mathbf{J}_q\mathbf{e} + \mathbf{c}$ is the vertically integrated net product.[19]

We can now compare expression (4) with the original system in Pasinetti (1973, equation (2.1), p. 4):

$$(\mathbf{I} - \mathbf{A}^{\ominus})\mathbf{X}(t) = \mathbf{Y}(t) \tag{5}$$

with $\mathbf{A}^{\ominus} = \mathbf{A}^{(C)} + \mathbf{A}^{(F)}\widehat{\boldsymbol{\delta}}, \quad \mathbf{A} = \mathbf{A}^{(C)} + \mathbf{A}^{(F)}$.

The first apparent difference is a consequence of introducing pure joint products. Specifically, the original identity matrix $\mathbf{I}$ in (5) is here replaced by the Make matrix $\mathbf{V}_q$ in (4). Secondly, matrix $\mathbf{A}^{\ominus}$, "representing that part of the initial stocks of [circulating and fixed] capital goods that are actually used up each year by the production process" (Pasinetti, 1973, p. 4) is given, in our formulation, by the matrix of replacement needs $\mathbf{R}_q$.[20]

The key difference, however, is that the original vector of gross outputs, $\mathbf{X}(t)$ in (5), is here replaced by the *observed* unitary vector of activity levels, $\mathbf{e}$ in (4). While in system (5) there is a clear-cut separation between current gross output levels and the technique in use, this is not possible in (4). In empirical Use Tables, any separation between activity levels and techniques involves an element of arbitrariness, given that current period matrices contain implicit growth (or decay) components, allowing for extended reproduction in following period(s).[21]

[19]Note that net product vector $\mathbf{y}$ differs from the traditional Input-Output vector of final demand domestically produced. Besides their coincidence as regards truly final uses (private and government consumption as well as exports), the latter includes gross fixed capital formation, whereas the former includes expansion requirements *only*, for *both* circulating and fixed capital inputs.

[20]Note that $\mathbf{A}^{(F)}\widehat{\boldsymbol{\delta}}$ is a matrix of depreciation charges obtained by applying given *a priori* commodity-specific (row-wise) depreciation quotas $\widehat{\boldsymbol{\delta}}$ to stock matrix $\mathbf{A}^{(F)}$. Instead, working with physical replacement needs avoids looking at fixed capital flows from the value added side.

[21]In fact, matrix $\mathbf{A}^{(C)}$ of circulating capital inputs per unit of gross output cannot be identified with $\mathbf{U}_q$ in (1), as the Use Table collected by statistical institutes "includes all non-durable goods and services with an expected life of less than one year which are used up in the process of production by industries" (EUROSTAT, 2008, p. 146) not only to reproduce past output levels, but to expand (or contract) productive capacity. While $\mathbf{A}^{(C)}$ describes a technique in use, $\mathbf{U}_q$ includes both a given technique and effects from changing activity levels. Crucially, given that production takes time, current inputs are met from past outputs, so observed matrices depend on activity levels of several periods. For a clarification of the relation between empirical magnitudes and theoretical concepts in dynamic Input-Output schemes, see Lager (2000, especially, pp. 248-251).

As a consequence, changes in empirical transaction matrices may be due to varying activity levels or on-going technical progress; the analytical distinction between growth and technical change need not be unique. Given the aim of building an objective scheme for productivity accounting, while we do not separate unitary input requirements from the level of operation of each industry, we do consider replacement and expansion components separately.

Turning back to (4), provided $(\mathbf{V}_q - \mathbf{R}_q)$ is non-singular, observed unitary activity levels $\mathbf{e}$ may be expressed as:

$$\mathbf{e} = (\mathbf{V}_q - \mathbf{R}_q)^{-1}\mathbf{y}$$

while activity level *indexes* for each vertically integrated sector $i = 1, \ldots, n$ are given by:

$$\mathbf{x}_\nu^{(i)} = (\mathbf{V}_q - \mathbf{R}_q)^{-1}\mathbf{y}^{(i)} \tag{6}$$

with $\mathbf{y}^{(i)} = \mathbf{e}_i y_i$, $\sum_i \mathbf{y}^{(i)} = \mathbf{y}$ and $\sum_i \mathbf{x}_\nu^{(i)} = \mathbf{e}$.

2.2. Vertically hyper-integrated sectors

Recovering data for computing vertically hyper-integrated sectors is more straightforward, in empirical terms, than for vertically integrated ones. Building growing sub-systems requires first a redefinition of the concept of net output, which includes demand for final commodities only, as given by vector $\mathbf{c}$ in our set-up. This implies that *gross* investments, and not only self-replacement requirements, are part of the means of production, and therefore there is no need to distinguish between replacement needs and new investments as we did in section 2.1. The key concept behind hyper-integration is the induced character of (fixed and circulating) investment expenditures.[22]

[22]Precisely, "[i]t is this *derived demand* aspect of investment goods, due to their being used as means of production, that is new and typical of production systems" (Pasinetti, 1981, p. 176).

In order to compute vertically hyper-integrated sectors, we depart from commodity balances (1), re-ordering terms to obtain:

$$(\mathbf{V}_q - \mathbf{U}_q - \mathbf{F}_{k_q})\mathbf{e} = \mathbf{c} \tag{7}$$

Pasinetti's (1988) original formulation of growing subsystems is given by:[23]

$$\mathbf{B}\mathbf{X}(t) - \mathbf{A}\mathbf{X}(t) - g\mathbf{A}\mathbf{X}(t) - \mathbf{A}\sum r_i \mathbf{X}^{(i)}(t) = \mathbf{C}(t) \tag{8}$$

To connect (7) and (8), note first Pasinetti's (1988) treatment of fixed capital as a joint product, in the tradition of Sraffa (1960). Instead, we adopt an empirically more tractable procedure, working with gross fixed capital flow and stock matrices, though incorporating pure joint products.

Also in this case, the vector of operation intensities $\mathbf{X}(t)$ in (8) is replaced by the unitary vector $\mathbf{e}$ in (7), while $\mathbf{B}\mathbf{X}(t)$ performs a similar role than $\mathbf{V}_q\mathbf{e}$.[24] The most apparent difference is represented by the fact that it is not possible to find a one-to-one correspondence between matrices $\mathbf{A}$, $g\mathbf{A}$, $\mathbf{A}\sum r_i \widehat{\mathbf{X}}^{(i)}(t)$ and matrices $\mathbf{U}_q$, $\mathbf{F}_{k_q}$; rather, $(\mathbf{U}_q + \mathbf{F}_{k_q})\mathbf{e}$ performs in empirical system (7) the role that $\mathbf{A}\mathbf{X}(t) + g\mathbf{A}\mathbf{X}(t) + \mathbf{A}\sum r_i \mathbf{X}^{(i)}(t)$ has in (8).[25] Finally, net output $\mathbf{C}(t)$ in (8) corresponds to vector $\mathbf{c}$ in (7).

There is an additional subtle but crucial point in comparing (7) and (8). The subsystem-specific expansion component $\mathbf{A}\sum r_i \mathbf{X}^{(i)}(t)$ implies that activity levels have to be solved separately for each vertically hyper-integrated sector in the theoretical formulation. This is

[23] See Pasinetti (1989, expression (2.1a), p. 479). We have slightly re-ordered terms in this presentation.

[24] Note, however, that output matrix $\mathbf{B}$ also includes 'old' durable instruments of production while Make matrix $\mathbf{V}_q$ includes only 'new' finished products.

[25] Two reasons motivate this: (i) while $\mathbf{A}\mathbf{X}(t)$ are self-replacement requirements and $g\mathbf{A}\mathbf{X}(t) + \mathbf{A}\sum r_i \mathbf{X}^{(i)}(t)$ correspond to expansion components, both of circulating and fixed capital without distinction, $\mathbf{U}_q$ and $\mathbf{F}_{k_q}$ separate between circulating and fixed capital inputs, respectively, but without splitting into replacement and expansion requirements; and (ii) while in the theoretical scheme there is a neat separation between technique (matrix $\mathbf{A}$) and activity levels, in empirical structures this is not attempted.

so because the analysis evaluates gross output requirements to satisfy a counter-factual (or normative) steady growth path in final consumption, in which specific demand expansion components (r_i) are given data. Instead, as our aim is to set up a productivity accounting framework based on *actually observed* period-by-period changes, it need not be true that intermediate transactions will have followed a commodity-specific steady growth path, conforming to the dynamics of net output.

Hence, in this *empirical* reconstruction of growing subsystems, we take changing activity levels for what they have been, building hyper-integrated sectors by re-proportioning observed unitary operation intensities in correspondence to each single component of net output vector $\mathbf{c}$.

To sum up, by comparing the load of assumptions and computations required to arrive at system (4) with respect to system (7), it emerges that observed statistical outlays are much more suitable for vertically hyper-integrated analyses than for vertically integrated ones. The key issue is that the separation between replacements and new investments in vertically integrated analyses — which is always subject to quite a high degree of arbitrariness — is not at all necessary in setting up growing subsystems, where matrices $\mathbf{U}_q$ and $\mathbf{F}_{k_q}$ include both self-replacement and expansion components. This purely computational advantage has, however, a deeper conceptual foundation:

> [w]ith technical change going on, each machine is never replaced by an exact similar physical machine, and this makes it impossible to say what is that is replaced and kept intact. Measurement in terms of units of [hyper-integrated] productive capacity overcomes this possibility.
>
> (Pasinetti, 1981, p. 178)

Turning back to (7), provided $(\mathbf{V}_q - \mathbf{U}_q - \mathbf{F}_{k_q})$ is non-singular, observed unitary activity levels $\mathbf{e}$ may be expressed as:

$$\mathbf{e} = (\mathbf{V}_q - \mathbf{U}_q - \mathbf{F}_{k_q})^{-1}\mathbf{c}$$

while activity level *indexes* for each vertically hyper-integrated sector $i = 1, \ldots, n$ are given by:

$$\mathbf{x}_\eta^{(i)} = (\mathbf{V}_q - \mathbf{U}_q - \mathbf{F}_{k_q})^{-1}\mathbf{c}^{(i)} \tag{9}$$

with $\mathbf{c}^{(i)} = \mathbf{e}_i c_i$, $\sum_i \mathbf{c}^{(i)} = \mathbf{c}$ and $\sum_i \mathbf{x}_\eta^{(i)} = \mathbf{e}$.

Obtaining (*net output, activity level indexes*) tuples at the vertically integrated and hyper-integrated levels — $(\mathbf{y}^{(i)}, \mathbf{x}_\nu^{(i)})$ from (6) and $(\mathbf{c}^{(i)}, \mathbf{x}_\eta^{(i)})$ from (9), respectively — is our departure point for setting up a productivity accounting scheme in disaggregated physical terms.

2.3. Direct, integrated and hyper-integrated labour requirements and productive capacities

In view of devising a framework exclusively based on observable magnitudes, the technical conditions of production may be summarised into labour requirements and stocks of fixed and circulating capital goods (i.e. productive capacity). In formal terms:

$$(\mathbf{l}^T, \mathbf{S}) = (\mathbf{l}^T, \mathbf{K}_q^* + \mathbf{R}_{u_q}^*) \tag{10}$$

$$(L, \mathbf{s}) = (\mathbf{l}^T\mathbf{e}, \mathbf{S}\mathbf{e}) \tag{11}$$

$$(L_i, \mathbf{s}_i) = (\mathbf{l}^T\mathbf{e}_i, \mathbf{S}\mathbf{e}_i) = (L_i \times 1, \mathbf{s}_i \times 1) \tag{12}$$

where $\mathbf{l}^T$ stands for the industry employment vector (in man-hours), and $\mathbf{S}$ contains fixed and circulating capital stocks (both domestically produced and imported) required to support the production of gross outputs $\mathbf{q}$, with each industry operating at observed unitary intensities $\mathbf{e}$. In (12), it is explicit that each *industry*'s employment (L_i) and commodity requirements ($\mathbf{s}_i$) are expressed per unitary operation intensity.

As has been pointed out by Kurz and Salvadori (1995, pp. 168-9), vertically integrated (and hyper-integrated, we may add) coefficients offer alternative descriptions of the technique in use. These alternatives, by shifting the disaggregated unit of analysis from the industry to the commodity subsystem, allow to perform a true price-volume separation, which is not possible otherwise.[26]

Conceptually, measuring the change in technical requirements for the reproduction of a given net output allows to account for circularity in production within each subsystem, while keeping a disaggregated physical dimension of technical progress. To see this point, consider the expressions corresponding to (10)-(12) at the vertically integrated and hyper-integrated levels, respectively:

$$(\boldsymbol{\nu}^T, \mathbf{H}) = (\mathbf{l}^T(\mathbf{V}_q - \mathbf{R}_q)^{-1}, \mathbf{S}(\mathbf{V}_q - \mathbf{R}_q)^{-1}) \tag{13}$$

$$(L, \mathbf{s}) = (\boldsymbol{\nu}^T\mathbf{y}, \mathbf{H}\mathbf{y}) \tag{14}$$

$$(L_\nu^{(i)}, \mathbf{s}_\nu^{(i)}) = (\boldsymbol{\nu}^T\mathbf{y}^{(i)}, \mathbf{H}\mathbf{y}^{(i)}) = (\boldsymbol{\nu}^T\mathbf{e}_i y_i, \mathbf{H}\mathbf{e}_i y_i) = (\nu_i y_i, \mathbf{h}_i y_i) \tag{15}$$

where $\boldsymbol{\nu}^T$ is the vector of vertically integrated labour coefficients, and $\mathbf{H}$ the matrix of vertically integrated productive capacity,[27] and:

$$(\boldsymbol{\eta}^T, \mathbf{M}) = (\mathbf{l}^T(\mathbf{V}_q - \mathbf{U}_q - \mathbf{F}_k)^{-1}, \mathbf{S}(\mathbf{V}_q - \mathbf{U}_q - \mathbf{F}_k)^{-1}) \tag{16}$$

$$(L, \mathbf{s}) = (\boldsymbol{\eta}^T\mathbf{c}, \mathbf{M}\mathbf{c}) \tag{17}$$

$$(L_\eta^{(i)}, \mathbf{s}_\eta^{(i)}) = (\boldsymbol{\eta}^T\mathbf{c}^{(i)}, \mathbf{M}\mathbf{c}^{(i)}) = (\boldsymbol{\eta}^T\mathbf{e}_i c_i, \mathbf{M}\mathbf{e}_i c_i) = (\eta_i c_i, \mathbf{m}_i c_i) \tag{18}$$

where $\boldsymbol{\eta}^T$ is the vector of vertically hyper-integrated labour coefficients, and $\mathbf{M}$ the matrix

[26]Note, in fact, that in the presence of pure joint products, any disaggregated industry magnitude (e.g. labour requirements) is usually computed with respect to gross output by industry, given by: $\mathbf{e}^T\widehat{\mathbf{p}}_s\mathbf{V}_q$, which necessarily involves a price aggregator (e.g. $\mathbf{p}_s$). This is not so when the analysis is kept at the commodity level.

[27]C.f. expressions (4.2b) and (4.3b) and the related interpretation of vertically integrated coefficients in Pasinetti (1973, p. 6).

of vertically hyper-integrated productive capacity.[28]

Expressions (10), (13) and (16) provide synthetic alternative descriptions of the technique in use, each referring to a different concept of output — unitary operation intensities in (10), consumption-*cum*-expansion requirements in (13), and consumption requirements only in (16) — though at the same time exhausting aggregate labour force and commodity requirements $(L, \mathbf{s})$, as can be seen from (11), (14) and (17).[29]

The key point of this comparison lies in industry or commodity-level expressions (12), (15) and (18). Productivity analysis should focus on measuring changing technical requirements per physical unit of *net* output, while for industry magnitudes (12) the separation between technique and unitary observed activity levels is not unique and, even if attempted, would assess labour and commodity requirements to reproduce *gross* output. Instead, the logical operation of vertical (hyper-)integration, as reflected in (15) and (18), allows to establish a clear-cut separation between technical requirements and net output, even at a disaggregated level.

In fact, total subsystem labour — $L_{\nu}^{(i)} = \nu_i y_i$ in (15) and $L_{\eta}^{(i)} = \eta_i c_i$ in (18) — is a scalar magnitude that, nevertheless, captures the whole intricate network of inter-industry relations through coefficients ν_i and η_i. This is clearly not the case with industry-level magnitudes. Moreover, when capital goods are measured in units of subsystem-specific productive capacity — i.e. in terms of composite commodities $\mathbf{h}_i$ in (15) and $\mathbf{m}_i$ in (18) — it is possible to distinguish, period-by-period, the required *pace* of capital accumulation (given by the dynamics of net output) from the changing *physical composition* of productive capacities (given by the evolution of vectors $\mathbf{h}_i$ and $\mathbf{m}_i$).[30]

Clearly, National Accounts do not provide physical, but nominal data. However, it can be

[28]C.f. expressions (2.9) and (2.10) and the related interpretation of vertically hyper-integrated coefficients in Pasinetti (1988, pp. 127-8).

[29]Note that each of these expressions provides an alternative *decomposition* of the same aggregates.

[30]See Pasinetti (1973, pp. 28-9) for a reflection on this latter point.

shown[31] that, since period-by-period magnitudes may be expressed at constant (or current and past-year) prices, computing variations through time of commodity-level variables in a pure joint products framework makes price effects to vanish. By proceeding in this way we obtain pure volume changes, which remain an essential pre-requisite for productivity accounting in physical terms.

3. Measures of changes in productivity and direction of technical change

Based on the alternative descriptions of the technique in use provided in Section 2.3, we compute vertically integrated and hyper-integrated measures of productivity changes and indexes of direction of technical change, both at the sectoral and aggregate level.[32]

3.1. Productivity Changes

Departing from (15), total labour productivity in each vertically integrated sector i is given by the ratio of net product to total labour requirements in the corresponding self-replacing subsystem:

$$\alpha_\nu^{(i)} = \frac{y_i}{L_\nu^{(i)}} = \frac{y_i}{\nu_i y_i} = \frac{1}{\nu_i} = \frac{1}{\mathbf{l}^T(\mathbf{V}_q - \mathbf{R}_q)^{-1}\mathbf{e}_i} \tag{19}$$

Note that $\alpha_\nu^{(i)}$ is not a 'partial' measure as, besides labour inputs, it takes into account changes in circulating and fixed capital requirements for self-replacement.

Correspondingly, total labour productivity in each vertically hyper-integrated sector i is given by the corresponding ratio of net product to subsystem labour requirements:

$$\alpha_\eta^{(i)} = \frac{c_i}{L_\eta^{(i)}} = \frac{c_i}{\eta_i c_i} = \frac{1}{\eta_i} = \frac{1}{\mathbf{l}^T\left(\mathbf{V}_q - \mathbf{U}_q - \mathbf{F}_{k_q}\right)^{-1}\mathbf{e}_i} \tag{20}$$

[31]See Appendix B for details.

[32]In order to ease exposition, we present all indicators in levels. As shown in Appendix B, computing changes of commodity-level magnitudes valued using the same price system cancels out the influence of prices.

But what is the rationale for formulating vertically hyper-integrated measures with respect to vertically integrated ones? As pointed out by Garbellini (2010, pp. 48-9), the key issue lies in the switch from static to dynamic analysis. The vertically integrated sector is an essentially static construct. Being new investments included in the net output of each vertically integrated sector, the part of it consisting in capital goods needs to be exchanged between (or redistributed among) subsystems for them to expand (or contract) their productive capacity — composite commodity $\mathbf{h}_i$ in (15). As a consequence, as soon as we consider the *evolution of subsystems through time*, they cease to be completely autonomous. Allowing for a true separation between changes in technique and dynamics of net output requires gross investments to be included among the means of production, as we did in (20) following Pasinetti (1988). Hence, hyper-integrated productivity measures reflect comprehensive though disaggregated surplus generating capacity in physical terms, within a set of subsystem-specific *expanding* circular flows.

To see this point, we may decompose changes in total labour requirements at the integrated and hyper-integrated levels from (19) and (20), respectively:[33]

$$\Delta\% L_{\nu}^{(i)} = \Delta\% y_i - \Delta\% \alpha_{\nu}^{(i)} \tag{21}$$

$$\Delta\% L_{\eta}^{(i)} = \Delta\% c_i - \Delta\% \alpha_{\eta}^{(i)} \tag{22}$$

Expression (21) is a 'spurious' decomposition: changes in y_i are due to changes in final demand for both consumption commodities and new investment goods. But the process of reproduction of capital goods is itself subject to technical change, so that changes in vertically integrated net output are also influenced by changes in productivity, i.e. by the second addendum of the decomposition. On the contrary, expression (22) correctly separates the effect of changes in the composition of effective demand for final uses from the effects of

[33]By $\Delta\% x$ we denote the rate of change between $t-1$ and t of variable x.

technical progress, thereby separating what does from what does not re-enter the circular flow.[34] Hence, it is $\Delta\%L_{\eta}^{(i)}$ that displays the structural dynamics of employment as intended by Pasinetti (1981, pp. 94-7).

On a first thought, the fact that vertically hyper-integrated labour productivity depends on the pattern of accumulation may be seen as a disadvantage, because productivity usually reflects the surplus generating capacity of an economy, independently of the use made of this social surplus (as new investment demand, for example).

However, if we adopt the view that economic systems are continuously undergoing growth (or decay), considering new investment as part of the means of production has the advantage of providing us with a notion of surplus that captures the truly *final* effects of technical progress. Its aggregate expected outcome should be increasing private consumption per capita in a closed economy without government, and not necessarily higher consumption-*cum*-investment per head, as in the vertically integrated case.

It must be noticed that, *ceteris paribus*, *higher* growth rates of final effective demand imply *lower* vertically hyper-integrated productivity *levels*. On this potential source of criticism, two points should be made.[35] First, if the exercise performed is one of comparative dynamics, the accumulation rates themselves are part of the data known or assumed. So productivity levels may be computed for every feasible growth path. When analysing productivity differences in a single economy this should suffice to discard the critique, because the analysis is conditional upon every feasible set of data. Second, the proviso *ceteris paribus* is surely not very realistic. A fast accumulating society embodying technical progress in new machines will not have the same direct labour input or material input requirements as a stagnant society with almost no new investment.

[34] See also Pasinetti (1986, p. 7) on the role of subsystems as "an analytical device that allows us to separate in an unambiguous way what pertains to the surplus from what pertains to the circular process".

[35] We would like to thank one anonymous referee for calling our attention on the need to consider more carefully the usefulness of hyper-integration for measuring productivity changes.

When performing instead an empirical exercise without assuming any *a priori* set of growth rates, total labour productivity differences may be explained by multiple underlying determinants. In this case, accelerated growth may imply a tendency towards increasing hyper-integrated labour content of commodities, as employment would be expected to rise. So it will be up to the decreasing nature of technical coefficients — either embodied in the capital inputs introduced through new investment or through the intensification of the division of labour — to act as counter-tendencies for an overall decreasing trend of labour content to manifest itself. Therefore, the final effect of these two countervailing forces is only reflected in an *hyper*-integrated productivity measure, not in a vertically integrated one.

Besides looking at the dynamics of labour productivity as a single magnitude, it is also informative to decompose each vertically hyper-integrated labour coefficient into its direct and (hyper-)indirect component. In order to do so, from the definition of $\boldsymbol{\eta}^T$ in (18), for each final commodity i we may write:

$$\eta_i = \boldsymbol{\eta}^T \mathbf{e}_i = \mathbf{l}^T \mathbf{V}_q^{-1} \mathbf{e}_i + \mathbf{l}^T \mathbf{V}_q^{-1} (\mathbf{U}_q + \mathbf{F}_{k_q})(\mathbf{V}_q - \mathbf{U}_q - \mathbf{F}_{k_q})^{-1} \mathbf{e}_i \tag{23}$$

where each addendum in the RHS of the equation may be defined as:

$$\eta_{dir}^{(i)} = \mathbf{l}^T \mathbf{V}_q^{-1} \mathbf{e}_i, \qquad \eta_{hyp}^{(i)} = \mathbf{l}^T \mathbf{V}_q^{-1} (\mathbf{U}_q + \mathbf{F}_{k_q})(\mathbf{V}_q - \mathbf{U}_q - \mathbf{F}_{k_q})^{-1} \mathbf{e}_i$$

The first addendum ($\eta_{dir}^{(i)}$) represents the direct labour employed by all industries producing commodity i, while the second ($\eta_{hyp}^{(i)}$) stands for the indirect and hyper-indirect labour requirements to reproduce a unit of commodity i for final uses. Accordingly, by defining $\omega_{\eta,d} = \eta_{dir}^{(i)}/\eta_i$ as the share of direct labour in vertically hyper-integrated labour per unit of net output, it is possible to assess the (relative) degree of interdependence of each subsystem (which will be given by $1 - \omega_{\eta,d}$).

Finally, to obtain a synthetic measure of total productivity changes at an aggregate level, we compute Pasinetti's (1981) *standard rate of growth of productivity*:[36]

$$\rho^* = \frac{\sum_{i=1}^{n} L_\eta^{(i)} \Delta\%\alpha_\eta^{(i)}}{\sum_{i=1}^{n} L_\eta^{(i)}} = \sum_{i=1}^{n} \frac{L_\eta^{(i)}}{L} \Delta\%\alpha_\eta^{(i)} \tag{24}$$

The standard rate of productivity growth consists of a weighted average of total labour productivity changes in every hyper-integrated sector, the weights being the ratio of total labour requirements in the corresponding growing subsystem to aggregate employment.

3.2. Direction of Technical Change

Replying to Solow (1957), Pasinetti (1959) advanced a methodological proposal to compute productivity changes and the direction of technical change in which the reproducible character of produced means of production was explicitly taken into account.[37]

In particular, Pasinetti (1959) focused his attention on the evolution of two ratios: Q/L and C/N, where Q is the quantity of final consumption commodity actually produced, C is the productive capacity necessary for reproducing Q, L is the (direct) labour employed in its production and N "can be interpreted as the quantity of labor which would be necessary for reproducing the existent capacity, with the technique available at the time observations are made"(Pasinetti, 1959, p. 273). While Q/L is the labour productivity in the production of net-output, C/N measures labour productivity in the reproduction of capacity.[38] Hence:

> A change through time of Q/L can be assumed by itself to be an indication of change in productivity only if C/N changes in the same proportion. If C/N does not change in the same proportion at least two parts of the change have to be distinguished — a neutral effect equal

[36] Note that Pasinetti (1981) advanced this measure in the context of a simplified description of the technique in use (in which there were no inter-industry relations). Instead, formula (24) provides a computable generalisation within an empirical Supply-Use framework with pure joint products, explicitly acknowledging its vertically hyper-integrated character.

[37] A detailed presentation of this debate is beyond the scope of this paper. The discussion between Pasinetti and Solow on this specific topic has re-emerged after the posthumous publication of a research paper by Stone (1998[1960]), which gave rise to further exchanges (Solow, 1998; Pasinetti, 1998).

[38] It should be clear that C/N measures a counter-factual, as N corresponds to a measure of *current* and *co-existing* labour, no reference at all being made to series of *dated* labour quantities.

to the proportional change of that ratio which has changed the least, and a labor saving effect — or alternatively a capital saving effect — given by the excess of the proportional change of Q/L over C/N — or alternatively of C/N over Q/L

(Pasinetti, 1959, p. 273)

Thus, by defining:

$$\beta = \frac{Q/L}{C/N} \tag{25}$$

we may assess the direction of technical change, according to the movement of β.

A key point of this formulation lies in measuring capital goods in units of capacity C currently required to reproduce a given (hyper-integrated) net-output Q. In fact, by adopting this unit of measurement for capital goods, $C = Q$ in every period and $\beta = N/L$.

Clearly, β was originally conceived in the context of an economy producing a single final commodity, without taking into account the complexities of inter-industry relations. However, as soon as general interdependence is accounted for, such an aggregate index could never be purely 'technical', as it would depend on compositional changes in the vector of final uses. At best, the original measure could be conceptually thought of as a subsystem-specific index. Moreover, this indicator should mirror the evolution of the capital/net-output ratio, reflecting the over-all capital intensity of the system, in the sense of Harrod (1948).

To translate the logic of Pasinetti's (1959) index β into the formulation for growing subsystems in (16)-(18), we need to establish a correspondence with Q, L, C, and N. When considered at the level of the single hyper-integrated sector, L may be associated with $L_{\eta}^{(i)}$ in (18), while Q and C correspond to c_i. Finally, we may define:

$$N_{\eta}^{(i)} = \boldsymbol{\eta}^T \mathbf{m}_i c_i \tag{26}$$

i.e. the quantity of *co-existing* vertically hyper-integrated labour that would be necessary for the reproduction of the existing productive capacity with the technique actually in use.

In this way, the disaggregated index for the direction of technical change in each vertically hyper-integrated sector i may be written as:

$$\beta^{(i)} = \frac{c_i / L_\eta^{(i)}}{c_i / N_\eta^{(i)}} = \frac{N_\eta^{(i)}}{L_\eta^{(i)}} = \frac{\boldsymbol{\eta}^T \mathbf{m}_i c_i}{\eta_i c_i} = \frac{\boldsymbol{\eta}^T \mathbf{m}_i}{\eta_i} \tag{27}$$

while the economy-wide index of capital intensity is given by:

$$\beta^* = \frac{Q/L}{C/N} = \frac{N}{L} = \frac{\sum_i N_\eta^{(i)}}{\sum_i L_\eta^{(i)}} = \frac{\boldsymbol{\eta}^T \mathbf{M} \mathbf{c}}{\boldsymbol{\eta}^T \mathbf{c}} \tag{28}$$

Note that the series of subsystem-specific indexes $\beta^{(i)}$ as well as the aggregate index β^* are 'pure numbers'.[39] Moreover, it is worth stressing that while β^* depends on the composition of final consumption $\mathbf{c}$ (its movement through time thus depending on compositional changes in net-output), sectoral indexes $\beta^{(i)}$ are intrinsically 'technical', since they are independent of the structure of final uses.[40] The intrinsically technical character of subsystem magnitudes with an over-all average that depends on the composition of final demand is also present in $\Delta\%\alpha_\eta^{(i)}$ and ρ^* (expressions (20) and (24) above, respectively).

4. An empirical exploration

In this section we present and discuss the results of computing sectoral and aggregate measures of productivity increase and direction of technical change, introduced in Section

[39] In fact, it is straightforward to show that the absolute level of both measures can be computed starting from nominal magnitudes, since the effect of prices cancels out:

$$\frac{\boldsymbol{\eta}^T \widehat{\mathbf{p}}_s^{-1} \widehat{\mathbf{p}}_s \mathbf{M} \widehat{\mathbf{p}}_s^{-1} \widehat{\mathbf{p}}_s \mathbf{c}}{\boldsymbol{\eta}^T \widehat{\mathbf{p}}_s^{-1} \widehat{\mathbf{p}}_s \mathbf{c}} = \frac{\boldsymbol{\eta}^T \mathbf{M} \mathbf{c}}{\boldsymbol{\eta}^T \mathbf{c}} = \beta^*$$

$$\frac{\boldsymbol{\eta}^T \widehat{\mathbf{p}}_s^{-1} \widehat{\mathbf{p}}_s \mathbf{m}_i p_i^{-1}}{\eta_i p_i^{-1}} = \frac{\boldsymbol{\eta}^T \mathbf{m}_i}{\eta_i} = \beta^{(i)}$$

[40] In this sense, while $\beta^{(i)}$ adequately reflects the direction of technical change in each growing subsystem, the interpretation of β^* as indicating the 'type' of technical change at an aggregate level is not warranted. See Pasinetti (1981, p. 214).

3, for the Italian case throughout 1999-2007. Yearly series of square 30×30 (commodity $\times$ industry) Supply-Use Tables at the 2-digit NACE Rev. 1 level, as well as gross fixed capital stock and flow matrices and labour input data have been obtained from the Italian National Institute of Statistics (ISTAT).[41]

4.1. Effect of pure joint products

A distinctive feature of the present study consists in taking a set of square commodity $\times$ industry Supply-Use Tables as the point of departure, instead of adopting an Input-Output technology assumption to obtain an 'industry' or 'commodity' model.[42] This is done not only for theoretical consistence with Classical production models,[43] but also because none of the main technology assumptions allows for *both* a genuine price-volume separation and an assured semi-positivity of the direct input requirements matrix.[44]

However, adopting a joint products framework prevents from obtaining 'well-behaved' systems so neatly as with single-product schemes. Theoretical models may overcome this limitation by assuming that the economy is 'all-productive', meaning that every commodity in the net-output is separately producible at non-negative activity levels[45] or, for the case of growing economies (at a uniform rate g), that the system is 'g-all-productive'.[46]

As regards vertical integration, 'all-productiveness' holds if and only if $(\mathbf{V}_q - \mathbf{R}_q)^{-1}$ is semi-positive. For the hyper-integrated case, we have not assumed uniform steady growth, so ready-made theoretical conditions do not strictly apply. However, 'g-all-productiveness' need hold if $(\mathbf{V}_q - \mathbf{U}_q - \mathbf{F}_{k_q})^{-1}$ is semi-positive.

[41] As regards particular characteristics of the dataset, as well as data preparation and estimation procedures, please refer to Wirkierman (2012, Appendix C).

[42] See EUROSTAT (2008, Chapter 11) for an exhaustive presentation of the four main Input-Output technology assumptions/transformation models: (a) commodity technology, (b) industry technology, (c) fixed industry sales structure and (d) fixed product sales structure.

[43] On theoretical grounds, Input-Output technology assumptions neglect joint products by re-allocating secondary production according to pre-defined criteria; see Lager (2011) for a detailed analysis.

[44] See Wirkierman (2012, Appendix B) for a formal analysis.

[45] Alternatively, "[a] system is *all-productive* if and only if all its subsystems have non-negative activity levels" (Schefold, 1989, p. 61).

[46] See Bidard (1996, p. 330) for sufficient conditions to guarantee 'g-all-productiveness'.

Unfortunately, as can be read from Table 1, the Italian economy (during 1999-2007) is neither all-productive nor g-all-productive, given that both inverse matrices contain some negative elements. But the fact of not possessing the (g-)all-productiveness property does not mean that empirical analyses are not meaningful. First, the fact that both $\boldsymbol{\nu}^T$ and $\boldsymbol{\eta}^T$ (i.e. the vectors of vertically integrated and hyper-integrated labour coefficients, respectively) do not contain any negatives is reassuring, in view of computing total labour productivity changes. Second, the few negative elements in matrix $\mathbf{M}$ of vertically hyper-integrated productive capacity correspond only to the Mining-Energy industry,[47] thus, sectoral indexes for the direction of technical change may be computed for all but one growing subsystem.[48]

[Table 1 here]

4.2. Distribution of the fruits of technical progress

Turning now to productivity accounting, Table 2 reports the aggregate dynamics of employment ($\Delta\%L$), average real wage rate ($\Delta\%(w/\overline{c}_p^*)$), productivity ($\rho^*$) and over-all capital intensity (β^*).

[Table 2 here]

A first remark concerns the extent to which productivity increases accrued to real wages. For the whole 1999-2007 period, ρ^* has exceeded $\Delta\%(w/\overline{c}_p^*)$ by a yearly average of 0.25 p.p., though it is interesting to notice that when productivity is falling (2000-2003), the real wage decreases to a lesser extent (their yearly average difference is -0.53 p.p.). Hence, productivity

[47]Specifically, in its interaction with Construction and Non-metallic minerals, and only between 2000 and 2002.

[48]Moreover, it can be informative to assess which commodities are separately producible at the vertically integrated or hyper-integrated level. Systems which are all-productive with respect to a subset of commodities are called 'partially all-productive' (Schefold, 1989, p. 196). In our case, it is noticeable that by just removing the negative entries of the Health services row in matrix $(\mathbf{V}_q - \mathbf{U}_q - \mathbf{F}_{k_q})^{-1}$, 14 out of 30 commodities would be separately producible at the hyper-integrated level. Note that negative entries never account for more than 4.7% of total yearly transactions.

movements amplify those of the real wage rate in both directions; however, the overall trend suggests that, on average, only 60% of productivity growth has accrued to wages; leaving real wages lagging behind productivity dynamics.

4.3. Substitution and degree of mechanisation

The link between increasing amounts of fixed capital inputs and technological unemployment is subtle and should be treated with care.[49] In the first place, as noted by Pasinetti (1981, Chapter IX), the ratio of fixed capital to labour should not be considered as an index of capital intensity but as an indicator of the degree of mechanisation of the system. Secondly, the substitution of capital for labour is not the consequence of changing relative 'factor' prices triggering movements along an isoquant, but is instead a *dynamic* process intimately connected to the hyper-integrated productivity growth of subsystems producing machinery with respect to the standard rate of productivity growth and the dynamics of the real wage rate. In fact:

> if, in any production process, at a certain point of time, machines are substituted for labour, the reason simply is that productivity in the machine producing sector is increasing faster than the over-all wage rate.
>
> (Pasinetti, 1981, p. 217)

In his assertion, Pasinetti (1981) is thinking in terms of a 'natural economic system', in which the average real wage increases precisely at rate ρ^*. However, the key point is the comparison between labour saving trends in subsystems producing fixed capital (mainly machinery) and over-all productivity growth (as measured by ρ^*), even if productivity increases do not fully accrue to wages (as has actually occurred in Italy between 1999 and 2007).

[49] This is clearly not the case in marginalist analyses of technical change, where the substitution mechanism conceives capital as an homogeneous quantity with a 'factor price' (the rate of profits), and suggests an inverse monotonic relation between the capital labour ratio and relative 'factor' prices (the ratio of the rate of profits to the real wage rate). See Pasinetti (1977) for a critique of the marginalist mechanism of 'factor input substitution'.

Intuitively, when productivity is increasing faster in the machinery subsystems than in the over-all economy, there is a comprehensive saving of labour in operating new machines with fewer workers with respect to using old durable instruments under current employment conditions.[50]

Hence, the frequently observed pattern of increasing degree of mechanisation reflects these labour saving trends, which may be only partially counter-balanced by an increase in effective demand for final commodities.[51]

By following the dynamics of hyper-subsystems DK and DL (corresponding to electrical and mechanical machinery products) in Table 2, an increasing degree of mechanisation in Italy is confirmed by the higher rate of productivity growth of the machinery complex (2.10 and 2.14 p.p.) with respect to both ρ^* and the real wage (0.66 and 0.37 p.p., respectively), on average.

4.4. Identification of dynamic subsystems

Table 3 reports sectoral output, employment and productivity changes at the direct, vertically integrated and hyper-integrated levels.

Given that productivity movements crucially depend on employment trends, it is useful to draw conclusions by looking at the joint dynamics of hyper-integrated labour productivity and total labour requirements, $\Delta\%\alpha_\eta^{(i)}$ and $\Delta\%L_\eta^{(i)}$, respectively. In fact, we have classified subsystems according to whether:

(i) Productivity growth is faster than average *and* subsystem labour is increasing ($\Delta\%\alpha_\eta^{(i)} > \rho^*$ and $\Delta\%L_\eta^{(i)} > 0$, respectively)

(ii) Productivity growth is faster than average *but* labour is being expelled from the subsystem ($\Delta\%\alpha_\eta^{(i)} > \rho^*$ and $\Delta\%L_\eta^{(i)} < 0$, respectively)

[50] Additionally, note that movements in the average wage rate rate will affect *all* subsystems, not only those producing final commodities; see Pasinetti (1981, p. 216, n. 28).

[51] An increase which, in turn, depends on the extent to which productivity increases accrue to real wages, allowing for higher real incomes and further final consumption.

(iii) Productivity growth is slower than average ($\Delta\%\alpha_{\eta}^{(i)} < \rho^*$)

[Table 3 here]

Group (i) includes *dynamic subsystems*, and mainly involves consolidated Italian manufacturing sectors like machinery and processed food products (DA, DL, DK, DM), diffused intermediates like plastics and metal products (DJ, DH), the pharma-health complex (DG, NN), together with logistics and financial services (II, JJ).

Group (ii) includes labour-expelling subsystems with faster than average productivity growth and mainly involves sectors producing inputs to the Construction industry — like Wood (DD), Furniture (the most important industry in DN) and Non-metallic minerals (DN, mainly cement) — and sectors in which international off-shoring and strong price competition has taken place (like Leather and Textiles). Note that in all these sectors hyper-integrated net-output dynamics — $\Delta\%c_i$ in column (06) — has been negative or almost nil.

Group (iii) is composed of subsystems lagging behind over-all productivity growth (ρ^*), mainly including the energy complex (CB, DF, EE) and two types of core service products: trade, accommodation, restaurants and business services (GG, HH, KK) on the one hand, and education and personal services (OO, PP, MM), on the other.

To have a quantitative idea of the relative weight of these subsystems in the economy, summing over column (09) within each category gives that groups (i), (ii) and (iii) represent, on average, about 32.74%, 19% and 48.26% of total labour requirements, respectively. Columns (06), (07), (08) capture the structural dynamics of employment (as obtained in expression (22) above). Labour expelled by subsystems in group (ii) has been, to a great extent, absorbed by service subsystems in group (iii) — HH, KK, OO, PP — as well as by mechanical machinery (DK), metal products (DJ), health services (NN), logistics (II) and finance (JJ) in group (i). While in the latter case demand dynamics — column (06) —

has more than counter-balanced labour saving trends, the opposite has occurred in sectors belonging to group (ii).

Note that a shift in the disaggregated unit of analysis from the industry to the growing subsystem leads to a complete change in some results. For example, from a comparison between $\Delta\%L_j$ in column (02) and $\Delta\%L_\eta^{(i)}$ in column (07), we have that while Chemicals, Plastics, Transport Equip. and Paper-Printing industries have expelled employment, the *growing* subsystems associated to their main product have instead absorbed labour. Interestingly, the dynamics associated to the relatively low share of direct in total labour — $\omega_{\eta,d}^{(i)}$ in column (10) — has been offset by indirect-*cum*-hyper-indirect dynamics.

However, a thorough inspection of columns (08) and (10) shows that no mechanical relationship can be established between the degree of interdependence of a subsystem (given by $1 - \omega_{\eta,d}^{(i)}$) and its productivity performance ($\Delta\%\alpha_\eta^{(i)}$). The same holds for the subsystem level of capital intensity — as 'proxied' by $\beta^{(i)}$ in column (11). In fact, note that some of the most capital intensive sectors produce Chemicals ($\beta^{(i)} = 9.38$), Public Administration ($\beta^{(i)} = 10.57$) and Business Services ($\beta^{(i)} = 16.02$), each of them belonging to a different group among (i)-(iii).

4.5. Direction of technical change and capital intensity

Focusing on the sectoral direction of technical change, i.e. the dynamics of $\beta^{(i)}$ in (27), we have that if $\Delta\%\beta^{(i)} > 0$, then $\Delta\%(c_i/L_\eta^{(i)}) > \Delta\%(c_i/N_\eta^{(i)})$, implying that total labour productivity increases faster than the reduction in labour content required to reproduce subsystem's i productive capacity. In terms of Pasinetti (1981, p. 209), this pattern corresponds to 'capital-intensity increasing' technical progress. In the case under study, it results from our computations that all growing subsystems but Education (MM) and Business Services (KK) follow this upward trend.[52]

[52]The values for $\Delta\%\beta^{(MM)}$ and $\Delta\%\beta^{(KK)}$ are -0.14% and -0.63%, on a yearly average basis, respectively.

By looking at column (11) in Table 3 the reader might wonder why we included average *levels* of $\beta^{(i)}$, instead of their rate of change. Being a pure number, $\beta^{(i)}$ might be directly compared across subsystems. In fact, in Section 3.2 we claimed that $\beta^{(i)}$ should *mirror* the capital/net-output ratio, i.e. the capital intensity, of each growing subsystem. In terms of statistically given (basic) prices $\mathbf{p}_s$, capital intensity in sector i is given by:

$$\kappa^{(i)} = \frac{\mathbf{p}_s^T \mathbf{M} \mathbf{c}^{(i)}}{\mathbf{p}_s^T \mathbf{c}^{(i)}} = \frac{\mathbf{p}_s^T \mathbf{m}_i c_i}{p_{s_i} c_i} = \frac{\mathbf{p}_s^T \mathbf{m}_i}{p_{s_i}} \tag{29}$$

whereas the economy-wide index is:

$$\kappa = \frac{\mathbf{p}_s^T \mathbf{M} \mathbf{c}}{\mathbf{p}_s^T \mathbf{c}} = \frac{\mathbf{p}_s^T \mathbf{S} \mathbf{e}}{\mathbf{e}^T \mathbf{f}_c} \tag{30}$$

A quick comparison of (29) and (30) with respect to (27) and (28), respectively, shows that the key difference lies in the use of a different set of weights to aggregate physical quantities: $\mathbf{p}_s^T$ in (29) and (30) as compared to $\boldsymbol{\eta}^T$ in (27) and (28). There is no *a priori* reason to expect that $\kappa^{(i)}$ is reflected in $\beta^{(i)}$ or, to control for the upward general trend in $\beta^{(i)}$, that $\beta^{(i)}/\beta^*$ mirrors $\kappa^{(i)}/\kappa$.

[**Figure 1 here**]

Figure 1 plots $\beta^{(i)}/\beta^*$ (x-axis) against $\kappa^{(i)}/\kappa$ (y-axis) for the year 2007.[53] Points (almost) lying on the dashed 45-degree line plot those subsystems whose deviation from the economy-wide capital intensity is (almost) correctly predicted by $\beta^{(i)}/\beta^*$ (examples are PP, MM, NN, II, LL; which are all service products). Below (above) the 45-degree line, $\beta^{(i)}/\beta^*$ over-estimates (under-estimates) $\kappa^{(i)}/\kappa$. With the exception of some outliers (subsystems AA, BB, KK, EE, DF, DG, JJ), conditional prediction of $\kappa^{(i)}/\kappa$ by $\beta^{(i)}/\beta^*$ is relatively accurate.[54]

[53] The main conclusions reached do not change by considering any other year between 1999 and 2006.

[54] Estimating a linear projection of $\beta^{(i)}/\beta^*$ on $\kappa^{(i)}/\kappa$ conditional on subsystem and year gives a multiple R^2

5. Concluding remarks

The aim of this paper has been to set up a physical productivity accounting scheme at a disaggregated level. To do so, we have relied on the notions of self-replacing and growing subsystems, rendered operational by means of vertical integration and hyper-integration, respectively. In particular, we established a precise correspondence between empirical categories of a set of Supply-Use Tables and the theoretical notions introduced by Pasinetti (1973, 1988). This allowed us to obtain alternative descriptions of the technique in use, and derive productivity indicators as well as indexes of direction of technical change.

While vertically integrated sectors have been frequently used in the literature, this is not the case for hyper-integrated sectors. It has been shown that each of these two notions requires a different concept of net product (none of which strictly coincides with the traditional Input-Output concept of final demand). Moreover, it has been argued that to accurately apply vertical integration, self-replacing requirements should be singled out, even though actual data include both self-replacement and expansion/contraction components. On the contrary, computation of vertically *hyper*-integrated sectors precisely requires these actual data. Hence, while growing subsystems are straightforward to obtain, self-replacing subsystems involve additional assumptions to empirically separate self-replacement from growth (or decay).

A crucial difference between this paper and other studies dealing with fixed capital inputs and vertical integration is that while in the latter depreciation matrices are considered a valid measure of self-replacement, we argue that a distinction between depreciation and physical replacements is essential. Depreciation pertains to the income (value-added) side of an Input-Output scheme, while physical replacements concern the expenditure side. Given that productivity accounting in physical terms should be always carried out departing from

of 0.9923, in which the unconditional mean (the intercept), $\beta^{(i)}/\beta^{*}$ and subsystem dummies are statistically significant (while year dummies are not). Results are available upon request.

the set of commodity balances by source of demand (i.e. the physical counterpart to the expenditure side), physical replacements, not depreciation, is the adequate magnitude to build subsystems for the measurement of technical change.

The fact of having directly departed from a set of Supply-Use Tables and gradually arrived at theoretical concepts allowed us to see that: (a) the separation between activity levels and techniques in empirically given structures involves an element of arbitrariness, and (b) a genuine price/volume-change separation cannot be obtained by applying any of the main Input-Output technology assumptions (while at the same time keeping a semi-positive direct input coefficients matrix). These two insights have important implications in the specification of a productivity accounting framework.

Empirically, the paper explored the case of the Italian economy during 1999-2007. By means of applying the set-up devised, it emerged that:

(a) only 60% of productivity growth has accrued to real wages, on average;
(b) the degree of mechanisation did increase;
(c) the most dynamic subsystems correspond to consolidated Italian sectors like machinery and processed food products, diffused intermediates like plastics and metal products, the pharma-health complex, together with logistics and financial services;
(d) technical change at the sectoral level has been (almost always) 'capital intensity increasing' in the sense of Pasinetti (1981, pp. 208-9).

Future research efforts should concentrate on a wider empirical application of the measures introduced (e.g. ρ^*, $\Delta\%\alpha_\eta^{(i)}$, $\beta^{(i)}$, β^*), in order to see how they perform in a long-period setting and across countries. Clearly, data availability restricts the realm of application, in particular due to a general lack of fixed capital stock and flow matrices, Use matrices for domestic output at basic prices, as well as the general difficulty of obtaining data in constant prices. But it is our firm conviction that only through the specification of an accurate theory of measurement it will be possible to obtain from statistical offices the elements required for the measurement of theory.

A. Matrix Notation

All throughout the paper, vectors are indicated by lower case boldface characters (e.g. $\mathbf{z}$), and have to be intended as column vectors unless explicitly transposed (e.g. $\mathbf{z}^T$); matrices are indicated by upper case boldface characters (e.g. $\mathbf{X}$), except for lower case characters with a hat (e.g. $\widehat{\mathbf{z}}$), indicating diagonal matrices with the vector elements on the main diagonal. Moreover, $\mathbf{e} = [1 \dots 1]^T$ is the sum vector and $\mathbf{e}_i = [0 \dots 1 \dots 0]^T$, with 1 in the i-th position, is a column selector vector.

In order not to make notation too heavy, we are going to distinguish a physical-quantity matrix from the corresponding one in nominal terms simply by adding the subscript $_q$; moreover, in general, almost all magnitudes will refer to time period t; in case of exceptions, the time lag i with respect to time period t will be indicated with the subscript $_{\pm i}$.[55] In the same way, all magnitudes without any special superscript will be intended as domestically produced ones; in all cases in which it will be necessary to refer to the imported component or to the sum of both domestically produced and imported components of a variable, we will do it by means of superscripts ^{m} and *, respectively.

The list of symbols composing the accounting framework introduced in Section 2 is reported in Table 4 below.

[Table 4 here]

B. Price-volume separation in empirical Supply-Use schemes

Consider a set of square $n \times n$ Supply-Use Tables, together with complementary gross fixed capital stock and flow matrices. In order to neatly separate price and volume components, we adopt the following working assumptions:

[55] So, for example, while matrix $\mathbf{V}$ will denote the make matrix, evaluated at current basic prices, in time period t, matrix $\mathbf{V}_{q(-1)}$ will denote the make matrix, in physical terms, referring to time period $t-1$.

(1) There is a uniform statistical *basic* price[56] for each domestically produced commodity (given by $\mathbf{p}_s$), as well as for each imported product (given by $\mathbf{p}_s^m$).

(2) The terms of trade $\widehat{\boldsymbol{\epsilon}}_p = \widehat{\mathbf{p}}_s^m \widehat{\mathbf{p}}_s^{-1}$ of the base-period are given.

Assumption (1) means that statistical basic prices used to construct Supply-Use Tables can be represented by diagonal matrices. This is justified in that, once taxes on products and trade and transport margins have been deducted, the price of a unit *at the factory* is equal for every unit produced, during the current accounting period.

Assumption (2) is required to work with total (domestically produced-*cum*-imported) magnitudes. As domestic and imported commodities have different price systems, in order to re-express nominal magnitudes with a different price-base, it is necessary to assume that the proportion between domestic and imported prices bears a given fixed relation, during the current accounting period.

Under these assumptions, Table 5 reports the price-volume decomposition of magnitudes composing the Supply-Use framework used throughout the analysis.

[Table 5 here]

Note that by expressing previous period magnitudes in current prices (e.g. $\widehat{\mathbf{p}}_s \mathbf{q}_{(-1)}$), activity level vector $\mathbf{x}_{(-1)}$ does not depend on relative prices. Moreover, every domestic-*cum*-imported magnitude (identified by superscript *) depends on the diagonal matrix of terms of trade $\widehat{\boldsymbol{\epsilon}}_p$.

[56] The sequence of prices in the SNA can be described as follows (UN, 2009, p. 103): Basic prices + Taxes on products excluding invoiced VAT - Subsidies on products = Producers' prices + VAT not deductible by the purchaser + Separately invoiced transport charges + Wholesalers' and retailers' margins = Purchasers' prices.

Hence, absolute magnitudes corresponding to the vertically integrated description of the technique in use that we can actually compute depend on statistical basic prices:

$$
\begin{aligned}
\mathbf{l}^T(\mathbf{V}-\mathbf{R}_u-\mathbf{R}_k)^{-1} &= \mathbf{l}^T(\widehat{\mathbf{p}}_s\mathbf{V}_q-\widehat{\mathbf{p}}_s\mathbf{R}_{u_q}-\widehat{\mathbf{p}}_s\mathbf{R}_{k_q})^{-1} = \\
&= \mathbf{l}^T(\mathbf{V}_q-\mathbf{R}_q)^{-1}\widehat{\mathbf{p}}_s^{-1} = \\
&= \boldsymbol{\nu}^T\widehat{\mathbf{p}}_s^{-1}
\end{aligned}
$$

for vertically integrated labour coefficients $\boldsymbol{\nu}^T$, and:

$$
\begin{aligned}
(\mathbf{K}^*+\mathbf{R}_u^*)(\mathbf{V}-\mathbf{R}_u-\mathbf{R}_k)^{-1} &= (\widehat{\mathbf{p}}_s\mathbf{K}_q^*+\widehat{\mathbf{p}}_s\mathbf{R}_{u_q}^*)(\widehat{\mathbf{p}}_s\mathbf{V}_q-\widehat{\mathbf{p}}_s\mathbf{R}_{u_q}-\widehat{\mathbf{p}}_s\mathbf{R}_{k_q})^{-1} = \\
&= \widehat{\mathbf{p}}_s(\mathbf{K}_q^*+\mathbf{R}_{u_q}^*)(\mathbf{V}_q-\mathbf{R}_{u_q}-\mathbf{R}_{k_q})^{-1}\widehat{\mathbf{p}}_s^{-1} = \\
&= \widehat{\mathbf{p}}_s\mathbf{S}(\mathbf{V}_q-\mathbf{R}_q)^{-1}\widehat{\mathbf{p}}_s^{-1} = \\
&= \widehat{\mathbf{p}}_s\mathbf{H}\widehat{\mathbf{p}}_s^{-1}
\end{aligned}
$$

for vertically integrated productive capacity matrix $\mathbf{H}$, as defined in (13).

However, this is not so for variations through time of vector $\boldsymbol{\nu}^T$ and for changes in the *units of* vertically integrated productive capacity — i.e. in the *columns* of matrix $\mathbf{H}$. By expressing all magnitudes in a common statistical price system and computing rates of change, the effect of prices vanishes:

$$
\begin{aligned}
&\left(\boldsymbol{\nu}^T\widehat{\mathbf{p}}_s^{-1}-\boldsymbol{\nu}_{(-1)}^T\widehat{\mathbf{p}}_s^{-1}\right)\widehat{\mathbf{p}}_s(\widehat{\boldsymbol{\nu}}_{(-1)})^{-1} = \left(\boldsymbol{\nu}^T-\boldsymbol{\nu}_{(-1)}^T\right)(\widehat{\boldsymbol{\nu}}_{(-1)})^{-1} \\
&p_j(\widehat{\mathbf{h}}_{j(-1)})^{-1}\widehat{\mathbf{p}}_s^{-1}\left(\widehat{\mathbf{p}}_s\mathbf{h}_j p_j^{-1}-\widehat{\mathbf{p}}_s\mathbf{h}_{j(-1)}p_j^{-1}\right) = (\widehat{\mathbf{h}}_{j(-1)})^{-1}\left(\mathbf{h}_j-\mathbf{h}_{j(-1)}\right)
\end{aligned}
$$

Proceeding in an analogous way for the vertically hyper-integrated case:

$$\begin{aligned}\mathbf{l}^T(\mathbf{V}-\mathbf{U}-\mathbf{F}_k)^{-1} &= \mathbf{l}^T(\widehat{\mathbf{p}}_s\mathbf{V}_q-\widehat{\mathbf{p}}_s\mathbf{U}_q-\widehat{\mathbf{p}}_s\mathbf{F}_{k_q})^{-1} = \\ &= \mathbf{l}^T(\mathbf{V}-\mathbf{U}_q-\mathbf{F}_k)^{-1}\widehat{\mathbf{p}}_s^{-1} = \\ &= \boldsymbol{\eta}^T\widehat{\mathbf{p}}_s^{-1}\end{aligned}$$

for vertically hyper-integrated labour coefficients $\boldsymbol{\eta}^T$, and:

$$\begin{aligned}(\mathbf{K}^*+\mathbf{R}_u^*)(\mathbf{V}-\mathbf{U}-\mathbf{F}_k)^{-1} &= (\widehat{\mathbf{p}}_s\mathbf{K}_q^*+\widehat{\mathbf{p}}_s\mathbf{R}_{u_q}^*)(\widehat{\mathbf{p}}_s\mathbf{V}_q-\widehat{\mathbf{p}}_s\mathbf{U}_q-\widehat{\mathbf{p}}_s\mathbf{F}_{k_q})^{-1} = \\ &= \widehat{\mathbf{p}}_s(\mathbf{K}_q^*+\mathbf{R}_{u_q}^*)(\mathbf{V}_q-\mathbf{U}_q-\mathbf{F}_{k_q})^{-1}\widehat{\mathbf{p}}_s^{-1} = \\ &= \mathbf{p}_s\mathbf{M}\widehat{\mathbf{p}}_s^{-1}\end{aligned}$$

for vertically hyper-integrated productive capacity matrix $\mathbf{M}$, as defined in (16).

Also in this case, computing changes through time allows us to get rid of price effects:

$$\left(\boldsymbol{\eta}^T\widehat{\mathbf{p}}_s^{-1}-\boldsymbol{\eta}_{(-1)}^T\widehat{\mathbf{p}}_s^{-1}\right)\widehat{\mathbf{p}}_s(\widehat{\boldsymbol{\eta}}_{(-1)})^{-1} = \left(\boldsymbol{\eta}^T-\boldsymbol{\eta}_{(-1)}^T\right)(\widehat{\boldsymbol{\eta}}_{(-1)})^{-1}$$

$$p_j(\widehat{\mathbf{m}}_{j(-1)})^{-1}\widehat{\mathbf{p}}_s^{-1}\left(\widehat{\mathbf{p}}_s\mathbf{m}_jp_j^{-1}-\widehat{\mathbf{p}}_s\mathbf{m}_{j(-1)}p_j^{-1}\right) = (\widehat{\mathbf{m}}_{j(-1)})^{-1}\left(\mathbf{m}_j-\mathbf{m}_{j(-1)}\right)$$

where $\mathbf{m}_j$ is the j-th column of matrix $\mathbf{M}$.

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Table 1: Negative elements in alternative descriptions of the technique in use

(In number of negative entries; yearly total transactions are $30^2 = 900$)

	1999	2000	2001	2002	2003	2004	2005	2006	2007
Inverse Matrices									
$(\mathbf{V}_q - \mathbf{R}_q)^{-1}$	n.a.	25	28	22	28	24	43	42	43
$(\mathbf{V}_q - \mathbf{U}_q - \mathbf{F}_{k_q})^{-1}$	20	23	20	19	20	22	38	39	43
Productive Capacities									
$\mathbf{H}$	n.a.	1	1	1	1	1	2	2	2
$\mathbf{M}$	n.a.	2	2	1	0	0	0	0	0
Labour content									
$\boldsymbol{\nu}^T$	n.a.	0	0	0	0	0	0	0	0
$\boldsymbol{\eta}^T$	0	0	0	0	0	0	0	0	0

Source: Own computation based on Supply-Use Tables (SUT) and National Accounts Data, ISTAT. Notes: ‘n.a.’ stands for not available, given that unavailable previous year data is required for current year computations.

Table 2: Aggregate Dynamics of Employment, Real Wage, Productivity and Capital Intensity in Italy (1999-2007)

(rates of change in p.p., mean values in average yearly p.p.)

	2000	2001	2002	2003	2004	2005	2006	2007	mean
Employment and Real Wage									
$\Delta\%L$	1.82	1.78	1.27	0.62	0.37	0.16	1.54	0.96	1.07
$\Delta\%(w/\overline{c}_p^*)$	-0.24	0.17	-0.31	-0.40	1.26	1.31	1.06	0.10	0.37
Productivity: Standard Rate and Machinery Subsystems									
ρ^*	2.54	-0.22	-1.23	-0.59	1.76	1.22	0.95	0.84	0.66
$\Delta\%\alpha_\eta^{(DK)}$	6.78	1.05	-3.99	0.35	6.02	1.37	4.19	1.04	2.10
$\Delta\%\alpha_\eta^{(DL)}$	7.24	-0.44	-1.06	-2.26	5.89	4.48	2.54	0.73	2.14
Over-all Capital Intensity Level									
β^*	6.37	6.43	6.58	6.68	6.66	6.80	6.77	6.88	6.65

Source: Own computation based on Supply-Use Tables (SUT) and National Accounts Data, ISTAT

Table 3: Dynamics of Output, Employment and Labour Productivity in Italy (1999-2007)

(mean values for period 1999-2007: rates of change in yearly average percentage points, levels in %)

	Direct		Integrated			Hyper-integrated					
	$\Delta\%q_i$	$\Delta\%L_j$	$\Delta\%y_i$	$\Delta\%L_{\nu}^{(i)}$	$\Delta\%\alpha_{\nu}^{(i)}$	$\Delta\%c_i$	$\Delta\%L_{\eta}^{(i)}$	$\Delta\%\alpha_{\eta}^{(i)}$	$\%L_{\eta}^{(i)}$	$\omega_{\eta,d}^{(i)}$	$\beta^{(i)}$
	(01)	(02)	(03)	(04)	(05)	(06)	(07)	(08)	(09)	(10)	(11)
	Dynamic Subsystems: $\Delta\%\alpha_{\eta}^{(i)} > \rho^*$ and $\Delta\%L_{\eta}^{(i)} > 0$										
DG:Chemicals	0.26	-0.45	2.48	0.33	2.31	3.59	0.43	3.40	1.43	27.40	9.38
DL:Electr. Machinery	2.19	1.13	0.28	-0.56	0.86	2.89	0.71	2.14	1.54	46.02	5.42
DA:Food-Tobacco	1.17	0.19	1.41	1.03	0.58	2.16	0.04	2.14	5.72	18.60	6.81
DK:Machinery n.e.c.	3.37	1.53	2.57	1.82	0.80	4.18	2.05	2.10	3.87	36.10	6.52
DH:Plastics	-0.07	-1.34	1.38	-0.07	1.47	2.73	0.77	1.96	0.67	40.11	7.51
JJ:Finance	3.70	0.87	2.80	1.60	1.10	3.65	1.79	1.79	1.58	50.00	6.63
II:Transport-Comm.	3.76	1.32	1.91	0.85	1.08	2.91	1.36	1.55	5.42	41.84	6.95
DM:Transport Equip.	0.92	-0.34	-0.31	-0.93	0.60	2.09	0.48	1.54	1.84	26.89	7.49
DE:Paper-Printing	1.52	-0.53	0.53	0.39	0.13	1.81	0.59	1.24	0.98	37.70	7.13
DJ:Metals	2.45	1.47	5.58	4.79	0.65	6.75	5.76	0.91	1.78	42.47	6.56
NN:Health	2.35	1.08	2.48	1.81	0.66	2.56	1.77	0.79	7.91	72.17	2.83
	Dynamic Productivity/Labour Expelling Subsystems: $\Delta\%\alpha_{\eta}^{(i)} > \rho^*$ and $\Delta\%L_{\eta}^{(i)} < 0$										
DD:Wood	0.25	-1.47	1.29	-1.05	1.78	0.09	-2.80	2.93	0.19	47.72	5.91
DC:Leather	-0.96	-2.78	-3.34	-3.28	0.05	-1.53	-3.59	2.16	1.32	38.05	5.69
DB:Textiles	-1.32	-2.41	-2.00	-1.86	-0.04	-0.58	-2.36	1.87	3.75	46.28	5.81
DN:Manufacture n.e.c.	0.19	-0.50	-2.23	-1.51	-0.60	-1.15	-2.53	1.43	1.92	41.56	5.80
LL:Public Admin.	1.24	-0.85	1.22	-0.34	1.58	1.25	-0.17	1.43	9.20	62.96	10.57
DI:Non-met. minerals	1.96	0.39	-1.53	-1.60	0.05	0.06	-0.96	1.08	0.70	38.25	7.25
AA:Agriculture	-0.34	-1.66	-1.29	-3.29	0.47	0.82	-0.05	0.88	1.92	70.89	5.45
	Productivity Lagging Subsystems: $\Delta\%\alpha_{\eta}^{(i)} < \rho^*$										
CB:Mining non-energy	-0.23	-1.85	0.78	3.68	-2.32	2.21	1.97	0.55	0.03	40.03	9.40
MM:Education	0.58	0.47	0.66	0.27	0.39	0.58	0.11	0.47	6.76	88.79	1.86
GG:Trade	1.70	0.69	0.76	0.85	-0.09	1.11	0.99	0.13	16.03	50.99	5.48
PP:Household Services	2.93	2.92	3.09	3.09	0.00	2.93	2.92	0.01	3.28	100.00	0.00
FF:Construction	2.40	3.03	1.33	2.12	-0.76	-0.61	-0.55	-0.06	0.63	50.67	4.35
HH:Hotel-Restaurant	1.68	2.56	0.90	2.40	-1.35	1.98	2.38	-0.37	7.87	57.38	4.69
EE:Energy	2.02	-1.38	1.12	1.72	-0.62	1.04	1.71	-0.46	0.73	20.89	15.80
BB:Fishing	-1.13	-0.36	-1.27	3.23	-3.39	-0.71	0.13	-0.80	0.19	81.64	2.80
OO:Personal Services	0.20	2.01	1.14	2.42	-1.11	1.50	3.10	-1.46	3.64	58.08	5.21
KK:Business Services	2.41	3.79	0.62	2.55	-1.87	1.42	2.93	-1.46	8.81	45.47	16.02
DF:Coke-Petroleum	0.03	0.08	-0.85	5.72	-4.48	-1.12	7.10	-5.59	0.29	18.44	11.55

Source: Own computation based on Supply-Use Tables (SUT) and National Accounts Data, ISTAT

Table 4: Symbols used throughout the paper

Symbol	Dimensions		Description
$\mathbf{U}$	$n \times n$	(commodity $\times$ activity)	Use matrix of circulating capital at basic prices
$\mathbf{V}$	$n \times n$	(commodity $\times$ activity)	Make matrix of gross outputs at basic prices
$\mathbf{F}_k$	$n \times n$	(commodity $\times$ activity)	gross fixed capital formation at basic prices
$\mathbf{f}_c$	$n \times 1$	(commodity $\times$ 1)	hyper-integrated net output at basic prices
$\mathbf{f}_{c_p}$	$n \times 1$	(commodity $\times$ 1)	final private consumption at basic prices
$\mathbf{f}_g$	$n \times 1$	(commodity $\times$ 1)	government consumption expenditure at basic prices
$\mathbf{f}_x$	$n \times 1$	(commodity $\times$ 1)	exports at basic FOB prices
$\mathbf{y}$	$n \times 1$	(commodity $\times$ 1)	vertically integrated net output in physical terms
$\mathbf{c}$	$n \times 1$	(commodity $\times$ 1)	hyper-integrated net output in physical terms
$\mathbf{q}$	$n \times 1$	(commodity $\times$ 1)	gross outputs in physical terms
$\mathbf{x}$	$n \times 1$	(activity $\times$ 1)	activity levels
$\mathbf{l}$	$n \times 1$	(activity $\times$ 1)	employment by industry (in number of persons engaged)
$\mathbf{K}$	$n \times n$	(commodity $\times$ activity)	gross fixed capital stocks at basic prices
$\mathbf{J}_k$	$n \times n$	(commodity $\times$ activity)	fixed capital new investments at basic prices
$\mathbf{J}_u$	$n \times n$	(commodity $\times$ activity)	circulating capital expansion at basic prices
$\mathbf{R}_k$	$n \times n$	(commodity $\times$ activity)	retirements of fixed capital inputs at basic prices
$\mathbf{R}_u$	$n \times n$	(commodity $\times$ activity)	circulating capital replacement needs at basic prices
$\mathbf{p}_s$	$n \times 1$	(commodity $\times$ 1)	statistical basic prices of domestically produced comm.
$\mathbf{p}_s^m$	$n \times 1$	(commodity $\times$ 1)	statistical prices of imported commodities

Table 5: Price-Volume Decomposition

Nominal	=	Price × Volume	Nominal	=	Price × Volume
$\mathbf{z}$	=	$\widehat{\mathbf{p}}_s\mathbf{q}$	$\mathbf{K}^*$	=	$\widehat{\mathbf{p}}_s\mathbf{K}_q^* = \widehat{\mathbf{p}}_s(\mathbf{K}_q + \widehat{\boldsymbol{\epsilon}}_p\mathbf{K}_q^m)$
$\mathbf{V}$	=	$\widehat{\mathbf{p}}_s\mathbf{V}_q$	$\mathbf{F}_k^*$	=	$\widehat{\mathbf{p}}_s\mathbf{F}_{k_q}^* = \widehat{\mathbf{p}}_s(\mathbf{F}_{k_q} + \widehat{\boldsymbol{\epsilon}}_p\mathbf{F}_{k_q}^m)$
$\mathbf{U}$	=	$\widehat{\mathbf{p}}_s\mathbf{U}_q$	$\mathbf{F}_k$	=	$\widehat{\boldsymbol{\theta}}\mathbf{F}_k^* = \widehat{\mathbf{p}}_s\widehat{\boldsymbol{\theta}}_q\mathbf{F}_{k_q}^* = \widehat{\mathbf{p}}_s\mathbf{F}_{k_q}$
$\mathbf{f}_c$	=	$\widehat{\mathbf{p}}_s\mathbf{c}$	$\mathbf{R}_k^*$	=	$\widehat{\mathbf{p}}_s\mathbf{R}_{k_q}^* = \widehat{\mathbf{p}}_s(\mathbf{R}_{k_q} + \widehat{\boldsymbol{\epsilon}}_p\mathbf{R}_{k_q}^m)$
$\mathbf{f}_k^*$	=	$\widehat{\mathbf{p}}_s\mathbf{f}_{k_q}^* = \widehat{\mathbf{p}}_s(\mathbf{f}_{k_q} + \widehat{\boldsymbol{\epsilon}}_p\mathbf{f}_{k_q}^m)$	$\mathbf{R}_k$	=	$\widehat{\boldsymbol{\theta}}\mathbf{R}_k^* = \widehat{\mathbf{p}}_s\widehat{\boldsymbol{\theta}}_q\mathbf{R}_{k_q}^* = \widehat{\mathbf{p}}_s\mathbf{R}_{k_q}$
$\mathbf{f}_k$	=	$\widehat{\mathbf{p}}_s\mathbf{f}_{k_q}$	$\mathbf{J}_k^*$	=	$\widehat{\mathbf{p}}_s(\mathbf{F}_{k_q}^* - \mathbf{R}_{k_q}^*)$
$\widehat{\boldsymbol{\theta}}$	=	$\widehat{\mathbf{p}}_s\widehat{\boldsymbol{\theta}}_q\widehat{\mathbf{p}}_s^{-1}$	$\mathbf{J}_k$	=	$\widehat{\boldsymbol{\theta}}\mathbf{J}_k^* = \widehat{\mathbf{p}}_s\widehat{\boldsymbol{\theta}}_q(\mathbf{F}_{k_q}^* - \mathbf{R}_{k_q}^*)$
$\mathbf{U}^*$	=	$\widehat{\mathbf{p}}_s\mathbf{U}_q^* = \widehat{\mathbf{p}}_s(\mathbf{U}_q + \widehat{\boldsymbol{\epsilon}}_p\mathbf{U}_q^m)$	$\mathbf{R}_u^*$	=	$\mathbf{U}^*\widehat{\mathbf{x}}_{(-1)} = \widehat{\mathbf{p}}_s\mathbf{U}_q^*\widehat{\mathbf{x}}_{(-1)} = \widehat{\mathbf{p}}_s\mathbf{R}_{u_q}^*$
$\mathbf{x}_{(-1)}$	=	$(\widehat{\mathbf{p}}_s\mathbf{V}_q)^{-1}\widehat{\mathbf{p}}_s\mathbf{q}_{(-1)} = \mathbf{V}_q^{-1}\mathbf{q}_{(-1)}$	$\mathbf{R}_u$	=	$\mathbf{U}\widehat{\mathbf{x}}_{(-1)} = \widehat{\mathbf{p}}_s\mathbf{U}_q\widehat{\mathbf{x}}_{(-1)} = \widehat{\mathbf{p}}_s\mathbf{R}_{u_q}$

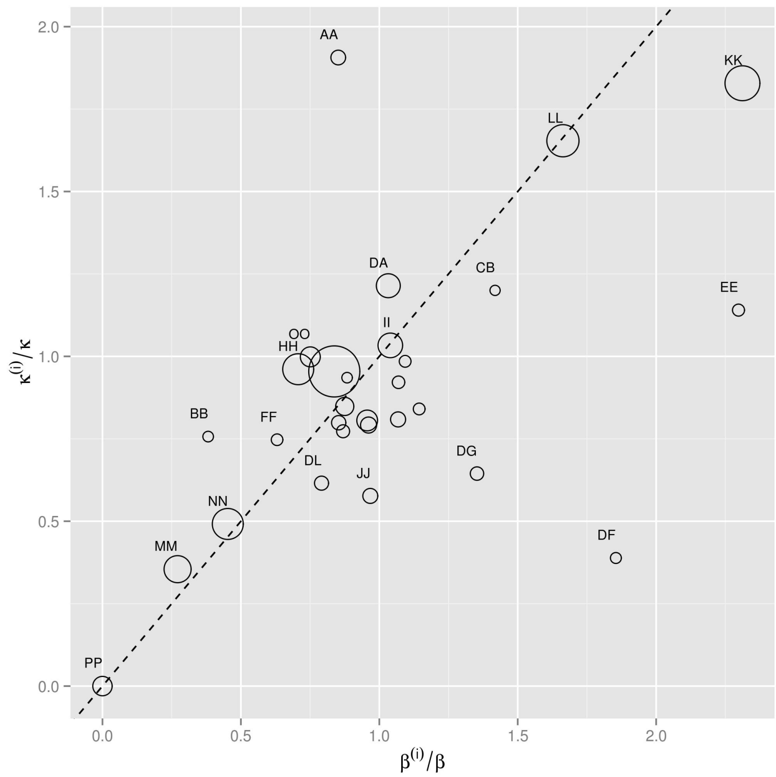

Figure 1: Deviation of Capital Intensity Indexes ($\beta^{(i)}$, $\kappa^{(i)}$) with respect to their economy-wide averages (β^{*}, κ), Italy, 2007. Circle size represents the weight of the hyper-subsystem in total employment ($L_{\eta}^{(i)}/L$). Source: Own computation based on Supply-Use Tables (SUT) and National Accounts Data, ISTAT