

Wave function of the radion with a bulk scalar field

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Abstract

The behaviour of the distance between two branes (the ‘radion’) in a braneworld model with a bulk scalar field is investigated. We show that the BPS conditions of supergravity ensure that the dynamics of the scalar field and the radion are not independent; we derive the four-dimensional effective action, showing that the effective theory is of scalar–tensor nature, coupling the radion to four-dimensional gravity.

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1 Introduction

The idea that we live on a hypersurface embedded in a higher–dimensional space sparked a lot of interest amongst cosmologists and particle physicists. It is very conceivable that the brane–world idea sheds light on some unsolved problems of our standard models of the micro– and macro–worlds. The idea is mainly motivated by recent progress in string– and M–theory [1], but phenomenological approaches led to important insights. In particular, the model by Randall and Sundrum plays the rôle of a simple playground where the brane world idea can be tested [2],[3]. However, as with all simple models, it might not be realized in nature. One important extension is to consider matter fields in the bulk, such as scalar fields (see [4]–[13] and references

therein). These matter fields exist in some fundamental approaches to brane worlds, such as heterotic M–theory [1] and supersymmetry [14], and could affect the brane matter fields, because of the coupling between them.

In this paper we discuss the dynamics of a brane-world scenario with two branes in the presence of a bulk scalar field. The second brane is separated from the first brane by a distance r_c . From the viewpoint of the four–dimensional theory the distance between the branes is a scalar degree of freedom, called the radion. The dynamics of the radion has been discussed in many papers (see [15]–[25] and references therein). In this paper we study in particular the coupling of the radion to the bulk scalar field. To do so we focus on linear perturbations around a static solution. In particular this involves solving the equations of motion for the scalar and gravitational perturbations. As usual we isolate two types of gravitational degrees of freedom: the first type comprises the gravitons and its imprints on the brane; the second type springs from the existence of two branes and leads to the radion modulus parameterizing the proper distance between the branes. In the case of the models considered here, the coupling between the bulk scalar field and the radion ensures that there is essentially only one scalar degree of freedom in the four–dimensional effective theory.

The paper is organized as follows: In section 2 we present the set–up of the theory and the corresponding graviton wave function. In section 3 we discuss the fluctuation of the distances between the branes and its coupling to the bulk scalar field. The effective low energy scalar-tensor theory is presented. We summarize our findings in section 4.

2 The Graviton and the Scalar Field Zero Mode

In this section we describe the brane world setup and describe the graviton and scalar field zero modes.

2.1 The Background Configurations

We consider a bulk Lagrangian consisting of two terms which describe gravity and the bulk scalar field, respectively:

$$S_{\text{bulk}} = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g_5} (R - \frac{3}{4}((\partial\phi)^2 + U)) \quad (1)$$

The boundary terms read

$$S_{\text{brane}} = -\frac{3}{2\kappa_5^2} \int d^5x \sqrt{-g_5} (\delta(z) - \delta(z - r_c)) U_B \quad (2)$$

where U_B is the superpotential related to the potential as

$$U = \left(\frac{\partial U_B}{\partial \phi} \right)^2 - U_B^2. \quad (3)$$

We will also include the Gibbons-Hawking term

$$S_{\text{GH}} = \frac{1}{\kappa_5^2} \int_{\delta} d^4x \sqrt{-g^B} K \quad (4)$$

where $\delta = \delta(z) - \delta(z - r_c)$ is the boundary of space-time while g^B is the boundary metric and K the extrinsic curvature. At the position of the branes we impose a Z_2 -symmetry.

In this paper we investigate solutions which are obtained from BPS-like equations, which are given by [4, 5]

$$\frac{a'}{a} = -\frac{U_B}{4}, \quad \phi' = \frac{\partial U_B}{\partial \phi} \quad (5)$$

where $' = d/dz$ for a metric of the form

$$ds^2 = dz^2 + a^2(z)\eta_{\mu\nu}dx^\mu dx^\nu. \quad (6)$$

We will particularly focus on the case where

$$U_B = 4ke^{\alpha\phi}, \quad (7)$$

with $\alpha = 1/\sqrt{3}, -1/\sqrt{12}$ which was obtained in a theory with supergravity in singular spaces [4]. In that case the solution reads

$$a(z) = (1 - 4k\alpha^2 z)^{\frac{1}{4\alpha^2}}, \quad (8)$$

whereas the scalar field solution is

$$\phi = -\frac{1}{\alpha} \ln(1 - 4k\alpha^2 z). \quad (9)$$

In the $\alpha \rightarrow 0$ we retrieve the AdS profile

$$a(z) = e^{-kz}. \quad (10)$$

Notice that in that case the scalar field decouples altogether. In this paper we will assume that the conditions (5) are valid everywhere.

2.2 Absence of the Scalar Field Zero Mode

We now consider perturbations around the background metric (6). In order to understand the origin of the radion mode, it was shown in [15] that it is essential to analyse the possible gauge choices which preserve the Gaussian normal coordinates (see also section 3 below). In this subsection we concentrate on the bulk scalar field perturbation and its transformation under infinitesimal coordinate transformations which preserve the Gaussian normal coordinate system.

Consider a brane located at

$$z = f(x^\mu) \quad (11)$$

where the bulk metric has the form (6), i.e. it reads

$$ds_0^2 = dz^2 + a^2(z)\eta_{\mu\nu}dx^\mu dx^\nu. \quad (12)$$

We consider perturbations of this background metric for which the perturbed metric has the form of a Gaussian normal coordinate system, i.e.

$$\delta g_{zz} = 0 = \delta g_{\mu z}. \quad (13)$$

When perturbing this background metric, one can change coordinates while preserving the Gaussian normal coordinate system and bringing the brane back to the origin of coordinates. This is achieved by considering vector fields ξ such that $z \rightarrow z + \xi^z$, $x^\mu \rightarrow x^\mu + \xi^\mu$ which satisfy

$$\xi^z \equiv f(x^\mu), \quad \xi^\mu = - \int \frac{dz}{a^2} \eta^{\mu\nu} \partial_\nu \xi_z + \eta^{\mu\nu} \epsilon_\nu(x^\mu). \quad (14)$$

Such a change of coordinates shifts the brane position to

$$z = 0. \quad (15)$$

We still have five gauge degrees of freedom, specified by ξ^z and ϵ^μ . In such a coordinate system the metric reads

$$ds^2 = ds_0^2 - \frac{U_B}{2} a^2(z) \xi_z dx^\mu dx_\mu - 2a^2 \left(\int \frac{dz}{a^2} \right) \partial_\mu \partial_\nu \xi_z dx^\mu dx^\nu \quad (16)$$

where we have put $\epsilon_\mu = 0$. Such a change of metric corresponds to gravitational modes which lead to the existence of a radion mode.

Let us now consider the case of a perturbed four dimensional part of the metric

$$ds^2 = dz^2 + a^2(z) dx^\mu dx_\mu + h_{\mu\nu} dx^\mu dx^\nu \quad (17)$$

when the brane is still located at $z = \xi_z$. In the Gaussian system at the origin we can write down the boundary conditions using

$$K_{\mu\nu} = \frac{1}{2} \partial_z g_{\mu\nu}^B \quad (18)$$

where g^B is the brane metric. The Israel junction conditions can be expressed as the identity

$$K_{\mu\nu} - g_{\mu\nu}^B K = \frac{1}{2} T_{\mu\nu}^B \quad (19)$$

evaluated on the brane. We have denoted by T^B the energy-momentum tensor on the brane.

In the coordinate system where the brane is at the origin, the perturbed metric reads

$$\hat{h}_{\mu\nu} = h_{\mu\nu} - \frac{U_B}{2} a^2(z) \xi_z \eta_{\mu\nu} - 2a^2 \left(\int \frac{dz}{a^2} \right) \partial_\mu \partial_\nu \xi_z, \quad (20)$$

while the perturbation of the scalar field is

$$\delta \hat{\phi} = \delta \phi + \frac{\partial U_B}{\partial \phi} \xi_z, \quad (21)$$

where $\delta \phi$ is the intrinsic part of the scalar field fluctuation as measured when the brane is at $z = \xi_z$. Notice that the perturbed metric and perturbed scalar field have two sources. The boundary condition evaluated at the origin leads to

$$\partial_z h_{\mu\nu}|_0 + \frac{U_B}{2} h_{\mu\nu}|_0 = 2\partial_\mu \partial_\nu \xi_z - \frac{a^2}{2} \frac{\partial U_B}{\partial \phi} \eta_{\mu\nu} \delta \phi \quad (22)$$

The scalar field boundary condition reads

$$\partial_z \delta\phi|_0 = \frac{\partial^2 U_B}{\partial\phi^2} \delta\phi \Big|_0 \quad (23)$$

These equations tell us that the brane fluctuation ξ_z acts as a source of gravitation on the brane.

Let us now describe the equations of motion in the bulk. We first concentrate on the Klein-Gordon equation

$$\square\delta\phi + \frac{1}{2}h'\phi' = \frac{1}{2}\frac{\partial^2 U}{\partial\phi^2}\delta\phi \quad (24)$$

where $h = h^\mu_\mu$. The Klein-Gordon equation admits a solution

$$\delta\phi(x^\mu, z) = \frac{\partial U_b}{\partial\phi} \hat{\phi}(x^\mu), \quad (25)$$

with

$$h = 0 \quad (26)$$

for any four dimensional massless $\hat{\phi}(x^\mu)$. It satisfies the boundary condition identically. Moreover we find that for these solutions the four dimensional metric perturbation is traceless. Of course, there might be other solutions for which the trace does not vanish; but this solution would not be the zero mode, which we consider here.

We are now going to analyse the Einstein equations to first order. They are best described by using the second order variation of the action. We consider the coordinate system where the brane is at $z = \xi_z$ and therefore write the metric perturbation as h_{ab} , where a, b is a five-dimensional index and $h_{55} = 0 = h_{\mu 5}$, as we consider Gaussian normal coordinates. The second order variation of the gravitational Lagrangian reads [26]

$$\begin{aligned} \delta^{(2)} S_g &= \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g_0} \left(\frac{1}{2} D_a h^{ab} D_c h^{cb} - \frac{1}{4} D_a h_{bc} D^a h^{bc} \right. \\ &\quad \left. - \frac{R}{4} h_{ab} h^{ab} + \frac{1}{2} R_{ab} h^a_c h^{bc} - \frac{1}{2} R_{abcd} h^{ad} h^{bc} \right) \end{aligned} \quad (27)$$

while the scalar Lagrangian gives

$$\delta^{(2)} S_\phi = -\frac{3}{8\kappa_5^2} \int \sqrt{-g_0} \left(\frac{\mathcal{L}_\phi}{3} h_{ab} h^{ab} + (\partial\delta\phi)^2 + \frac{1}{2} \frac{\partial^2 U}{\partial\phi^2} \delta\phi^2 + 2h^{ab} \partial_a \delta\phi \partial_b \phi \right) \quad (28)$$

where

$$\mathcal{L}_\phi = -\frac{3}{4} \left[(\partial\phi_0)^2 + U \right] \quad (29)$$

Notice that there is a contribution to the gravitational part from the transverse part of the gravitational field. We can simplify these actions using Einstein's equations

$$R_{ab} = T_{ab} - \frac{1}{3} T g_{ab} \quad (30)$$

and

$$R_{acbd} = -\frac{U_B^2}{16} (g_{ab}^0 g_{cd}^0 - g_{ad}^0 g_{cb}^0) + \frac{1}{4} \left(\frac{\partial U_B}{\partial \phi} \right)^2 (p_a g_{[cd}^0 p_{b]} - p_c g_{a[d}^0 p_{b]}) \quad (31)$$

where $p_a = \delta_{a5}$. The bulk Einstein equations are then

$$\square h_{ab} + \frac{U_B^2}{8} h_{ab} - D_{\{a} D_c h_{b\}}^c = \frac{3}{4} \partial_{\{a} \delta \phi \partial_{b\}} \phi_0 \quad (32)$$

Notice the unconventional appearance of the transverse part of the metric perturbation.

Let us focus on the (55) component of the Einstein equations. It leads to

$$\partial_z \delta \phi = 0 \quad (33)$$

This implies that $\delta \phi$ is a function of x^μ only. Now this is not compatible with the solution of the Klein-Gordon equation unless

$$\delta \phi \equiv 0. \quad (34)$$

Therefore the scalar field perturbation vanishes altogether in the coordinate system where the brane position fluctuates according to ξ_z .

Finally, the $(\mu 5)$ -component of the Einstein equations gives

$$D_\nu h^\nu_\mu = 0. \quad (35)$$

The metric perturbation is therefore found to be transverse-traceless. This will be used in the following section to identify the graviton wave-function.

In the case where the model with a bulk scalar field is embedded in supergravity in singular spaces one can understand the absence of an independent scalar degree of freedom as follows. At low energy the fifth component of the graviphoton completes the radion field to pertain to a chiral supermultiplet. No extra scalar degree of freedom is available to be paired with the scalar field in order to form the scalar part of a chiral supermultiplet. This implies that no low energy degree of freedom can be ascribed to the bulk scalar field.

2.3 The Graviton

Now we consider the gravitational equation in order to discuss the graviton modes. The field equation for the graviton simplifies to

$$\square h_{\mu\nu} + \frac{U_B^2}{8} h_{\mu\nu} = 0 \quad (36)$$

together with the boundary condition

$$\partial_z h_{\mu\nu}|_0 + \frac{U_B}{2} h_{\mu\nu}|_0 = 0 \quad (37)$$

where we impose $\xi_z = 0$. We decompose the solution into a product of a z -dependent part and a x^μ -dependent part:

$$h_{\mu\nu} = g(z) \chi_{\mu\nu}(x^\mu). \quad (38)$$

The Laplacian operator acting on the gravitational perturbation can be obtained by writing

$$\Gamma_{5\beta}^\alpha = -\frac{U_B}{4}\eta_\beta^\alpha, \quad \Gamma_{\alpha\beta}^5 = \frac{U_B}{4}g_{\alpha\beta}^0, \quad (39)$$

where the BPS conditions (5) have been used. This leads to

$$\square h_{\mu\nu} = (\partial_z^2 g)\chi_{\mu\nu} + g\square\chi_{\mu\nu} \quad (40)$$

and eventually

$$\square\chi_{\mu\nu} = a^{-2}\square^{(4)}\chi_{\mu\nu} + \left(\left(\frac{\partial U_B}{\partial\phi} \right)^2 - \frac{3}{8}U_B^2 \right) \chi_{\mu\nu} \quad (41)$$

We consider the zero modes of the gravitational perturbation only, i.e. we select

$$\square^{(4)}\chi_{\mu\nu} = 0 \quad (42)$$

implying that the graviton profile g satisfies

$$\partial_z^2 g + \left(\frac{1}{2} \left(\frac{\partial U_B}{\partial\phi} \right)^2 - \frac{1}{4}U_B^2 \right) g = 0. \quad (43)$$

This is a Schrödinger equation with a non-trivial potential. In the Randall-Sundrum case the brane potential is constant $U_B = 4k$ leading to

$$g_{RS} = a^2 \quad (44)$$

which satisfies the boundary condition.

To find the solution in the general case it is convenient to define the conformal coordinate

$$du = \frac{dz}{a}; \quad (45)$$

with such a choice and putting

$$g = \sqrt{a}\psi \quad (46)$$

we obtain the graviton equation

$$-\frac{d^2\psi}{du^2} + \frac{1}{a^{3/2}}\frac{d^2 a^{3/2}}{du^2}\psi = 0. \quad (47)$$

This is of the form of a supersymmetric quantum mechanics problem with

$$Q^\dagger Q\psi = 0 \quad (48)$$

where

$$Q = -\frac{d}{du} + \frac{d \ln a^{3/2}}{du}, \quad Q^\dagger = \frac{d}{du} + \frac{d \ln a^{3/2}}{du} \quad (49)$$

The two zero modes are easily found to be

$$\psi_1 = a^{3/2} \quad (50)$$

and

$$\psi_2 = a^{3/2} \int \frac{du}{a^3}. \quad (51)$$

The boundary condition can be reexpressed in terms of ψ

$$\frac{d(\psi a^{-3/2})}{du} = 0 \quad (52)$$

implying that the graviton profile is given by ψ_1 , i.e. that

$$g(z) = a^2 \quad (53)$$

Notice that this is the generalization to models with a bulk scalar field of the Randall-Sundrum result. The graviton is then described by

$$h_{\mu\nu} = a^2 \chi_{\mu\nu} \quad (54)$$

where $\chi_{\mu\nu}$ is a four-dimensional massless field. Now notice that we can use the residual gauge freedom specified by $\epsilon_\mu(x)$ in order to retrieve the fact that $\chi_{\mu\nu}$ possesses two independent polarizations only and therefore qualifies as the four dimensional graviton. This result guarantees that the low energy effective action will be a scalar-tensor theory. We are now going to discuss the appearance of an independent scalar mode whose coupling to gravity will be studied.

3 The Radion

So far we have discussed transverse-traceless modes with $\xi_z = 0$. In this section we include fluctuations of the brane positions and discuss their dynamics.

3.1 Brane Fluctuations

Coming back to the gravitational equation (32), we can investigate modes with $\xi_z \neq 0$. The boundary condition for such modes reads

$$\partial_z h_{\mu\nu}|_0 + \frac{U_B}{2} h_{\mu\nu}|_0 = 2\partial_\mu \partial_\nu \xi_z \quad (55)$$

Taking the trace of the boundary condition and using the tracelessness of the gravitational perturbation $h_{\mu\nu}$ we find

$$\square^{(4)} \xi_z \equiv \eta^{\mu\nu} \partial_\mu \partial_\nu \xi_z = 0. \quad (56)$$

This equation implies that the brane fluctuation corresponds to a massless four-dimensional field. It is a mode which complements the graviton and leads to a scalar-tensor theory at low energy. One can solve the gravitational equation with the Ansatz

$$h_{\mu\nu} = a^{1/2} \psi_\xi(z) \partial_\mu \partial_\nu \xi_z \quad (57)$$

where ψ_ξ satisfies (47). The boundary condition reads

$$\left. \frac{d(\psi_\xi a^{-3/2})}{du} \right|_0 = \left. \frac{2}{a} \right|_0 \quad (58)$$

One can identify the solution with

$$\psi_\xi = 2a_0^2 \psi_2(z) \quad (59)$$

leading to

$$h_{\mu\nu} = 2a_0^2 a^2 \left(\int \frac{dz}{a^4} \right) \partial_\mu \partial_\nu \xi_z \quad (60)$$

It is conspicuous that the two zero modes of the gravitational equation are transcribed in the two types of physical modes, the graviton and the brane fluctuation.

3.2 Including a Second Brane

So far, we have not used the existence of the second brane. However, as we shall see in the following, it is important to include a second brane in this setting. The rôle of the second brane is two-fold. On the one hand, in a consistent supersymmetric setting, the existence of the second brane is crucial in order to preserve supersymmetry. On the other hand the second brane screens off the naked singularity from the first brane [7].

The metric tensor which is a solution of the gravitational equation can be rewritten as

$$\hat{h}_{\mu\nu} = a^2 \chi_{\mu\nu} + 2a_0^2 a^2 \left(\int \frac{dz}{a^4} \right) \partial_\mu \partial_\nu \xi_z - 2a^2 \left(\int \frac{dz}{a^2} \right) \partial_\mu \partial_\nu \xi_z - \frac{U_B}{2} a^2 \xi_z \eta_{\mu\nu} \quad (61)$$

where one recognizes the graviton and the brane fluctuation. One can define such a solution in the vicinity of each brane. In the following we shall denote by a_+ and ξ_z^+ the scale factor and the brane fluctuation on the positive tension brane (respectively a_- and ξ_z^- on the negative tension brane). In each patch surrounding each brane the metric tensor is single-valued. We still have to construct a metric tensor which is single-valued throughout the bulk.

To do so we first impose that the brane fluctuation solution of (47) is single valued. This is achieved by requiring

$$a_+^2 \xi_z^+ \equiv a_-^2 \xi_z^- \equiv \xi \quad (62)$$

where ξ is a four dimensional massless field. Notice that this mode is intrinsically non-local relating the fluctuations on one brane to the fluctuations on the other brane. The metric tensor is now

$$\hat{h}_{\mu\nu}^+ = a^2 \xi_{\mu\nu} + 2a^2 \left(\int \frac{dz}{a^4} \right) \partial_\mu \partial_\nu \xi - 2 \frac{a^2}{a_+^2} \left(\int \frac{dz}{a^2} \right) \partial_\mu \partial_\nu \xi - \frac{U_B}{2} \frac{a^2}{a_+^2} \xi \eta_{\mu\nu} \quad (63)$$

in the patch surrounding the positive tension brane. In the second patch close to the negative tension brane, the metric tensor is similar

$$\hat{h}_{\mu\nu}^- = a^2 \xi_{\mu\nu} + 2a^2 \left(\int \frac{dz}{a^4} \right) \partial_\mu \partial_\nu \xi - 2 \frac{a^2}{a_-^2} \left(\int \frac{dz}{a^2} \right) \partial_\mu \partial_\nu \xi - \frac{U_B}{2} \frac{a^2}{a_-^2} \xi \eta_{\mu\nu} \quad (64)$$

Obviously the metric tensor cannot be patched up in this form as the two expressions in the two patches are different.

However one can perform a change of coordinates which preserves the Gaussian normal coordinates and translates the second brane. In the second patch it reads

$$z \rightarrow z - \left(\frac{1}{a_-^2} - \frac{1}{a_+^2} \right) \xi \quad (65)$$

which transforms $h_{\mu\nu}^-$ into $h_{\mu\nu}^+$. The negative tension brane sits now at

$$T(x) = r_c - \left(\frac{1}{a_-^2} - \frac{1}{a_+^2} \right) \xi. \quad (66)$$

Now the bulk metric is single-valued and given by (63). Notice that the position of the second brane fluctuates according to fluctuations of the massless field ξ . Moreover the field $T(x)$ can now be interpreted as the radion measuring the size of the extra dimension. The radion is also a four dimensional massless field. In particular the radion measures the brane bending of the second brane.

We would like to obtain the metric tensor in a form where the size of the interval does not fluctuate, i.e. the radion would appear as a fluctuation of the metric. This can be achieved thanks to a new change of coordinates defined by

$$z \rightarrow z + \left(\frac{1}{a^2} - \frac{1}{a_+^2} \right) \xi \quad (67)$$

which rescales the size of the bulk in such a way that the second brane sits at $z = r_c$. Imposing that no (μz) component appears in the metric tensor implies that one must also transform

$$x^\mu \rightarrow x^\mu + \xi^\mu \quad (68)$$

where

$$\xi_\mu = - \frac{a^2}{a_+^2} \int dz \left(\frac{a_+^2}{a^4} - \frac{1}{a^2} \right) \partial_\mu \xi \quad (69)$$

This implies that the metric tensor becomes

$$\hat{h}_{\mu\nu} = a^2 \chi_{\mu\nu} - \frac{U_B}{2} \frac{a^2}{a_+^2} \xi \eta_{\mu\nu} \quad (70)$$

Let us now write the full metric including both the background and the perturbation

$$\begin{aligned} ds^2 &= a^2 \left(z + \left(\frac{1}{a^2} - \frac{1}{a_+^2} \right) \xi \right) dx^\mu dx_\mu \\ &+ \left(1 + \partial_z \left(\frac{1}{a^2} - \frac{1}{a_+^2} \right) \xi \right)^2 dz^2 + \left(a^2 \chi_{\mu\nu} - \frac{U_B}{2} \frac{a^2}{a_+^2} \xi \eta_{\mu\nu} \right) dx^\mu dx^\nu \end{aligned} \quad (71)$$

This can be simplified to

$$ds^2 = a^2(G(z, x))g_{\mu\nu}dx^\mu dx^\nu + (\partial_z G)^2 dz^2 \quad (72)$$

to linear order. We have defined

$$G(z, x) = z + \frac{\xi}{a^2}, g_{\mu\nu} = \eta_{\mu\nu} + \chi_{\mu\nu} \quad (73)$$

This is the generalization of the metric Ansatz proposed by Charmousis, Gregory and Rubakov [15].

Let us now derive the fluctuation of the scalar field in the various coordinate systems. When the two branes are translated we have $\delta\phi = 0$. Shifting the first brane to the origin implies that

$$\delta\phi^+ = \frac{U'_B}{a_+^2}\xi \quad (74)$$

in the first patch and

$$\delta\phi^- = \frac{U'_B}{a_-^2}\xi \quad (75)$$

in the second patch. Let us now perform the local change of coordinates in the second patch (65). This defines a single-valued scalar field fluctuation which coincides with $\delta\phi^+$ throughout the bulk. The change of coordinates (67) leads to a single-valued fluctuation

$$\delta\phi = U'_B \frac{\xi}{a^2} \quad (76)$$

Combined with the background scalar field this leads to

$$\phi(z, x) = \phi_0(G(z, x)) \quad (77)$$

Notice that the scalar field is now a function of the brane fluctuation parameter ξ , i.e. the radion. There is no scalar field dynamics *per se*.

3.3 The Scalar-Tensor Theory at Low Energy

Now that we have described the independent low energy degrees of freedom of the theory with a bulk scalar field we are in a position to discuss the low energy scalar tensor theory. We will only examine two relevant terms, i.e. the Einstein-Hilbert term and the radion kinetic term.

The Einstein-Hilbert kinetic term can be seen to be

$$\int d^4x \sqrt{-g} \Phi R \quad (78)$$

where $g_{\mu\nu}$ is the four dimensional metric and the effective Newton constant is

$$\Phi = \frac{1}{\kappa_5^2} \int_{\xi/a_+^2}^{r_c + \xi/a^2} a^2(y) dy \quad (79)$$

Notice the explicit dependence on the radion field. Similarly the kinetic terms for the radion field is

$$-\frac{3}{8\kappa_5^2} \int d^4x \sqrt{-g} K (\partial\xi)^2, \quad (80)$$

where

$$K = \int_0^{r_c} dz a^{-2} \left[U_B^2 + 2 \left(\frac{\partial U_B}{\partial \phi} \right)^2 \right]. \quad (81)$$

Notice that this is always positive.

Let us focus on the AdS_5 case where the scalar tensor theory is specified by

$$\Phi = \frac{1}{2k\kappa_5^2} (e^{-k\xi/a_+^2} - e^{-kr_c - k\xi/a_-^2}) \quad (82)$$

and

$$K = 8k(e^{2kr_c} - 1) \quad (83)$$

In the large r_c limit and for small fluctuations of the radion ξ this result coincides with the results of Garriga and Tanaka [29] on the Brans-Dicke parameter when coupling the theory to matter. For the general case the discussion of the coupling to matter for brane world models with a bulk scalar is left for a companion paper.

4 Conclusions

In this paper we have given a detailed discussion about the dynamics of a two-brane system with bulk scalar field. We have investigated the coupling of the distance between the branes, which is a scalar degree of freedom (radion) and four-dimensional gravity. When the BPS condition (5) is imposed on the system, we find that the bulk scalar field and the radion combine to give one independent scalar degree of freedom; i.e. the bulk scalar field does not give rise to an additional degree of freedom. At low energy we find that the theory reduces to a scalar-tensor theory coupling the radion to gravity.

There is one final step to be taken, namely the breaking of supersymmetry on the brane, before we are in a position to confront the brane world with a bulk scalar field with observation. It is likely that breaking supersymmetry will induce a potential term for the radion field in the low energy effective action. This is in progress.

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